Horizontal Product Differentiation: Disclosure and Competition

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This Version

Abstract

The unravelling argument in the disclosure literature established that when firms produce different qualities that are unknown to consumers, firms have an incentive to disclose this private information. Recent literature has established that this argument does not carry over to an environment where a monopoly firm produces a variety of horizontally differentiated products. This paper argues that the results of the recent literature are due to the assumption of a monopoly firm. We consider a horizontally differentiated duopoly market and show that all equilibria of the disclosure game have firms fully disclosing the variety they produce.

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1 Introduction

In a large number of markets, sellers have important information about product attributes that are not publicly observable. In many instances, however, firms have the option of voluntarily disclosing this information in a credible and verifiable manner through a variety of means such as independent third party certification, labeling, rating by industry associations (or government agencies) and through informative advertising.

There is a large literature dealing with the question whether firms have appropriate incentives to disclose information about the product they produce. Most of this literature deals with this issue in the context of vertical product differentiation, where different firms sell different qualities. In this context, the well-known unraveling argument,[1] establishes that a firm whose product is actually better than the average has a positive incentive to voluntarily disclose the quality of its product to buyers. This then induces every firm whose quality is above the average *undisclosed* quality to also disclose. The unraveling argument results in a situation where all private information about quality should be revealed through voluntary disclosure. Observed nondisclosure is then explained in terms of "disclosure frictions", such as disclosure costs, consumers not understanding the information that is disclosed, etc. Alternatively, Janssen and Roy (2010) show that nondisclosure can also be explained by a combination of *market competition* and the availability of *signaling* as an alternative means (to disclosure) of communicating private information.

Recently, Sun (2011) and Celik (2011) have analyzed the incentives for firms to disclose their product characteristics when horizontal product differentiation is the only or main dimension of differentiation. Both papers are set in a monopoly context. Sun (2011) shows that seller types with unfavorable horizontal attributes (towards the extreme points of the product line) do not have an incentive to disclose. In combination with vertical differentiation, her results imply that if either full disclosure of both attributes or no disclosure at all are the only possible reporting strategies, a seller with private information about both horizontal and vertical attributes may not want to disclose quality even if it is high. Celik (2011) shows that the amount of information disclosure critically depends on the strength of the buyer's preference for her ideal attribute. If buyers have very strong preferences for particular product varieties, then there exists an equilibrium in which the seller fully reveals variety. Otherwise, the seller only partially reveals the variety he produces. Moreover, the set of fully revealed locations monotonically shrinks from all to (almost) none as the buyer's preference for her ideal taste becomes weaker.

[1] See, Viscusi (1978), Grossman (1981), Milgrom (1981), Jovanovic (1982).

In this paper, we show that these results for horizontal product differentiation do not extend to a more competitive environment. In particular, we show in a duopoly set-up that full disclosure is always an equilibrium, and moreover, that there does not exist any equilibrium where firms do not fully disclose their product information. As there can be many messages with which firms fully disclose their information, the equilibrium strategies are not unique, but the equilibrium outcome of full disclosure is.

The model we consider has two firms located on a Hotelling line, where each particular location represents the variety of the product. Location is known to both firms, but not to consumers.[2] The two firms first simultaneously choose a message about their location. We assume that firms cannot lie. That is, the true location should be consistent with the message that is chosen. One way to think about this grain of truth assumption is that information is verifiable and that there is a large fine for providing information that turns out to be false. The assumption is in line with regulations concerning advertisement or other disclosure mechanisms requiring that firms provide truthful information. Firms can either send a rather vague message, indicating that their location is somewhere on the product line, as one extreme, or a much more precise message, indicating the precise location, as the other extreme, or anything in between. After firms have sent their messages, they both simultaneously choose prices. Consumers have quadratic transportation costs and decide where to buy the product after observing the messages and the prices. Given the information they receive, consumers update their beliefs about the location of the two firms and buy from the firm where the sum of transportation costs and price is the lowest.

The reason why in this environment all equilibria must be fully revealing is intimately related to the reason why in the standard Hotelling model with location choice, firms want to maximally differentiate from each other. Suppose that a firm would not choose a fully revealing strategy and would choose the same message for different locations. As the price cannot be used to signal the location, consumers will then be uncertain about the true location of the firm. In this case, the updated beliefs of consumers will be such that they do not assign full probability mass to the extreme locations that send this particular message. At least one of these extreme locations have then an incentive to deviate for two reasons. First, by fully revealing its location, a firm can reduce the uncertainty concerning the location for consumers and with convex transportation costs any

[2] The competitive disclosure literature can be divided into a stream that considers markets where firms know each others' type (see, e.g., Daughety and Reinganum (2007), Caldieraro, Shin and Stivers (2008) and Janssen and Roy (2011)) and another literature where they do not (see, e.g., Board (2009) and Hotz and Xiao (2011)). The first type of literature, and thus this paper, is relevant for markets where firms are active for some time and have the ability (and due to the frequent interaction also the incentives) to find out the product produced by a competitor.

reduction in uncertainty increases demand *ceteris paribus*.[3] Second, by having a perceived location that is further away from the competitor, firms charge higher price in the pricing game and this price effect outperforms the direct demand effect.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 describes the results and Section 4 concludes.

2 Model

Consider a horizontally differentiated duopoly, where the variety produced by each firm is represented by a particular location on the unit interval. Let x_i denote the variety produced by firm i and $x_i \in [0, 1]$, $i = 1, 2$. We focus on the disclosure policy of the firms and consider these varieties to be given for the firms. In the following, x_1 and x_2 will be referred to as locations or types of firms 1 and 2, respectively. We consider markets where firms know each others's location, but consumers are unaware of the specific location of firms. One way to think about this is that it requires resources to research the product characteristics of a firm and that rival firms are better equipped or have more incentives to do this than consumers. Production costs do not depend on firms' locations and without loss of generality are set to be equal to zero.

The demand side of the economy is represented by a continuum of consumers. Each consumer has a preference for the ideal variety of the good that she would like to buy, denoted by λ. The value of λ, or consumers' location on $[0, 1]$, follows a uniform distribution.[4] A consumer's net utility from buying variety x_i at price P_i, $i = 1, 2$, is $v - t(\lambda - x_i)^2 - P_i$, where v is the gross utility of a consumer when the variety of the good, x_i, matches with her ideal variety, λ, perfectly (i.e., when $x_i = \lambda$) and t measures the degree of disutility a consumer incurs when x_i and λ differ from each other. We assume that v is sufficiently large so that the market is fully covered. Each consumer then chooses to buy the good from the firm where her expected utility is maximized. The consumer has unit demand and if she buys from firm i, then firm i's payoff from the transaction is P_i; otherwise, the payoff of firm i is zero.

The timing of the game is as follows. At stage 0, Nature *independently* selects locations x_1 for firm 1 and x_2 for firm 2 from a strictly positive density function $f(x)$. The locations are known to both firms, but not to consumers. At stage 1, firms send a costless message $M_i \subset [0, 1]$, $i = 1, 2$, about

[3]One way to interpret this is that convex transportation cost introduces an element of risk aversion in consumers' preferences: *ceteris paribus* a consumer rather buys at a known location than at an unknown location with the same expected value.

[4]This specification with a continuum of consumers whose preferences for variety is uniformly distributed over the unit interval, is identical to the specification with a single consumer who has a privately known taste for a variety drawn from the uniform density function defined over $[0, 1]$.

their location, where $M_i = [0, 1]$ can be interpreted as "no message at all", or full non-disclosure of information by firm i. Messages have to contain a grain of truth in the sense that $x_i \in M_i$ for $i = 1, 2$. That is, firms cannot lie about their location. In the following we will refer to this assumption as the grain of truth assumption. At stage 2, firms simultaneously set prices. Finally, at stage 3, consumers observe the messages and the prices of the two firms and decide where to buy. At the end of the game, the payoffs of all players – firms and consumers – are realized. All aspects of the game are common knowledge.

Two important observations are in order at this point. First, the quadratic term in the utility function of consumers (the transportation costs) implies risk aversion with respect to x_i. That is, a consumer dislikes uncertainty about the variety of the good and given two messages with the same conditional mean, favors the one with the smaller variance. Second, even though consumers are assumed to have unit demand for the good, the probability of a purchase from a given firm declines with its price so that the expected demand function faced by each firm is downward sloping.

To proceed with the more formal analysis, we define the strategy spaces as follows. The reporting strategy of firm i is denoted by $m_i(x_i, x_j)$. The image of m_i belongs to all subsets of $[0, 1]$ such that $x_i \in m_i$. The pricing strategy of firm i is denoted by $p_i(x_i, x_j|M_i, M_j)$ where the messages sent by the two firms are M_i and M_j, respectively. Similarly, let the vector $b(\lambda, M_i, M_j, P_i, P_j)$ describe the buying strategy of a consumer with preferred variety λ, where $b = (1, 0)$ if the consumer buys the good from firm 1 and $b = (0, 1)$ if the consumer buys the good from firm 2. Finally, let $\mu_i(z|M_i, M_j, P_i, P_j)$ be the probability density that consumers assign to $x_i = z$ when the firms send messages M_i, M_j and set prices P_i, P_j. Note that at the moment when consumers have to decide from which firm to buy, they form beliefs μ_i not only on the basis of the observed messages and prices, but also on the basis of the equilibrium strategies, that is, equilibrium messages and prices, $m_i^*(x_i, x_j)$ and $p_i^*(x_i, x_j|M_i, M_j)$. All consumers process the information received in the same way and therefore have symmetric beliefs.

Before providing the details of the equilibrium notion which we will use to analyze the game, we consider the decision making of a consumer. To do so, we first find the ideal variety, $\widehat{\lambda}$, of the *indifferent* consumer, who obtains the same expected net utility of buying from either of the two firms, given the observed set of messages and prices. Then all consumers with ideal varieties below $\widehat{\lambda}$ buy from the firm with the most left perceived location and all others buy from the other firm. Therefore, $\widehat{\lambda}$ determines the expected demand faced by each firm and allows describing optimal prices and messages chosen by the firms at the previous two stages of the game.

Given the updated beliefs, the ideal variety $\widehat{\lambda}$ of the indifferent consumer is defined by the equality between the expected net utility of buying from firm 1 and the expected net utility of buying from firm 2:

$$v - tE\left((\widehat{\lambda} - x_1)^2 | \mu_1\right) - P_1 = v - tE\left((\widehat{\lambda} - x_2)^2 | \mu_2\right) - P_2 \tag{2.1}$$

In this expression, $tE\left((\widehat{\lambda} - x_i)^2 | \mu_i\right)$, $i = 1, 2$, is the expectation of the transportation costs of the indifferent consumer associated with buying from firm i, conditional on consumers' beliefs.

We solve this equality for $\widehat{\lambda}$. Notice that

$$\begin{aligned} E\left((\widehat{\lambda} - x_i)^2 | \mu_i\right) &= \widehat{\lambda}^2 + E\left(x_i^2 | \mu_i\right) - 2\widehat{\lambda} E\left(x_i | \mu_i\right) \\ &= \widehat{\lambda}^2 + var\left(x_i | \mu_i\right) + E^2\left(x_i | \mu_i\right) - 2\widehat{\lambda} E\left(x_i | \mu_i\right) \end{aligned}$$

so that (2.1) becomes:

$$\begin{aligned} \widehat{\lambda}^2 + var\left(x_1 | \mu_1\right) &+ E^2\left(x_1 | \mu_1\right) - 2\widehat{\lambda} E\left(x_1 | \mu_1\right) + \frac{P_1}{t} = \\ &= \widehat{\lambda}^2 + var\left(x_2 | \mu_2\right) + E^2\left(x_2 | \mu_2\right) - 2\widehat{\lambda} E\left(x_2 | \mu_2\right) + \frac{P_2}{t}. \end{aligned}$$

Thus, the ideal variety of the indifferent consumer is equal to:

$$\widehat{\lambda} = \frac{1}{2} \frac{\frac{P_2 - P_1}{t} + var\left(x_2 | \mu_2\right) - var\left(x_1 | \mu_1\right)}{E\left(x_2 | \mu_2\right) - E\left(x_1 | \mu_1\right)} + \frac{1}{2}\left(E\left(x_1 | \mu_1\right) + E\left(x_2 | \mu_2\right)\right) \tag{2.2}$$

This result has an immediate implication for the form of the expected demand functions of firms 1 and 2. In fact, since consumers with preferred variety $\lambda < \widehat{\lambda}$ ($\lambda > \widehat{\lambda}$) buy from the firm with the most left (right) perceived location and since the value of consumer's best-preferred variety is distributed uniformly over $[0, 1]$, $\widehat{\lambda}$ is also the value of the expected demand faced by the firm with the most left perceived location, given the prices P_1 and P_2 and the messages M_1 and M_2. Accordingly, $1 - \widehat{\lambda}$, the remaining share of the market, is the expected demand of the other firm. Without loss of generality, throughout the paper we consider that $E\left(x_1 | \mu_1\right) \leq E\left(x_2 | \mu_2\right)$. In case of strict inequality, $\widehat{\lambda}$ is the expected demand of firm 1 and $1 - \widehat{\lambda}$ is the expected demand of firm 2. The case when $E\left(x_1 | \mu_1\right) = E\left(x_2 | \mu_2\right)$ will be addressed separately later.

The derivation of expected demand helps to define the equilibrium notion we use. From (2.2) it follows that apart from the price difference, a firm's expected demand and, hence, its profit only depends on *expected* locations and on the precision of the messages about these locations, but (and this is important) *not* on actual locations. This is true both on the equilibrium and off the equilibrium path. That is, any type of firm that sends the same equilibrium message concerning location has equal incentives to set any out-of-equilibrium price. Given this fact, price cannot reasonably act as a signal of location.

A similar argument applies to the inability of firms to signal the location of their competitor. Note that as firms know their own location and the location of their competitor, a firm's type is, in principle, a two-dimensional object (x_1, x_2). Therefore, consumers could, in principle, make some inference on the location of the competitor upon observing a firm's message. Given the above, this, however, would not be a reasonable inference. Consider that, all types of a firm that have sent the same equilibrium message concerning location and that therefore are believed to have an identical location along the equilibrium path have equal incentives to make consumers believe that their competitor has a certain location. Moreover, equilibrium pay-offs of all types that could – *given the grain of truth assumption* – have sent the same equilibrium message, should receive the same equilibrium pay-off as otherwise one of the types should have an incentive to send another message. Therefore, if consumers would infer a certain location of the competitor after observing a particular out-of-equilibrium message, either all types (x_1, x_2) of the firm with the same x_1 component would want to deviate to that out-of-equilibrium message or no type would. But then it would be unreasonable for consumers to discriminate between firm types that differ only in the location of the competitor.

In principle, the same could apply to a firm trying to signal its own location. But here is where the grain of truth assumption becomes relevant. If a firm would deviate to a very precise message, then because of the grain of truth assumption only few (or in the limit, no) other types with different own location can imitate that signal. Thus, the grain of truth assumption makes it possible for out-of-equilibrium messages to signal some information about own location.

These considerations imply that in the context of our model where profits are only governed by prices and expected locations (and not by real locations), it is reasonable to confine attention to equilibria where consumer beliefs concerning a firm's location only depend on its own message and not on pricing decisions. Also, due to the grain of truth assumption consumers interpret the one dimensional message M_i of firm i as being uninformative about the location of the competitor. We call such a perfect Bayesian equilibrium a stable belief equilibrium and it is defined as follows.

Definition A stable belief *equilibrium is* a set of reporting and pricing strategies $m_1^*, m_2^*, p_1^*, p_2^*$ of the two firms, strategy b^* of a consumer, and the probability density functions π_1^*, π_2^* which satisfy the following conditions:

(1) For all M_1, M_2, P_1 and P_2, b^* is a consumer's best buying decision as defined below:

$$b(\lambda, M_1, M_2, P_1, P_2) = \\ = \begin{cases} (1,0) & \text{if } \int_0^1 (v - t(\lambda - x_1)^2 - P_1)\mu_1(x_1|M_1, M_2, P_1, P_2)dx_1 \geq \\ & \quad \int_0^1 (v - t(\lambda - x_2)^2 - P_2)\mu_2(x_2|M_1, M_2, P_1, P_2)dx_2 \\ (0,1) & \text{if } \int_0^1 (v - t(\lambda - x_2)^2 - P_2)\mu_2(x_2|M_1, M_2, P_1, P_2)dx_2 \geq \\ & \quad \int_0^1 (v - t(\lambda - x_1)^2 - P_1)\mu_1(x_1|M_1, M_2, P_1, P_2)dx_1 \end{cases} \tag{2.3}$$

(2) Given (1) and given the messages sent by the two firms and the price set by the competitor, p_i^* is the price that maximizes the expected profit of firm i, $i = 1, 2$.

(3) Given (1), (2) and given the message sent by the competitor, m_i^* is the message that maximizes the expected profit of firm i, $i = 1, 2,$, subject to the constraint that $x_i \in m_i$.

(4) For all M_1, M_2, P_1 and P_2, a consumer updates his or her beliefs in the following way:[5]

$$\mu_i(z|M_1, M_2, P_1, P_2) = \begin{cases} \frac{f(z)}{\int_{z \in M_i} f(z)dz} & \text{on the equilibrium path} \\ \text{any beliefs satisfying the property} & \\ \mu_i(z|M_i, M_j, P_1, P_2) = \mu_i(z|M_i, M_j', P_1', P_2') & \\ \text{for any } P_1, P_2, P_1', P_2', M_j, M_j' & \text{off the equilibrium path} \end{cases}$$

Part (1) of the definition states that for any observed messages and prices, a consumer buys a unit of the product from the firm, where her expected net utility, given the updated beliefs, is maximized. Each firm rationally anticipates the best response of consumers to any given messages and prices, and chooses the price and message that maximize its expected profit. This is stated in parts (2) and (3). Finally, part (4) claims that consumers update beliefs about the locations using Bayes' rule for any M_1, M_2, P_1 and P_2 that occur with positive density along the equilibrium path and that, as discussed above, off-the-equilibrium path beliefs about firm i's location cannot depend on prices and the message sent by the other firms as any type of firm has equal incentives to choose any price. Given this independence of beliefs from prices and the message of the rival firm (on and off the equilibrium path), prices and the message of the other firm can be omitted from the notation for beliefs, leading to $\mu_i(z|M_i)$.

[5]Note that Bayes' rule cannot be applied when M_i or a subset of M_i is discrete. For example, if $M_i = \{y, z\}$, then both events, $x_i = y$ and $x_i = z$ have ex-ante zero probability. In this case, updating proceeds as follows:

$$\mu_i(z|M_1, M_2, P_1, P_2) = \lim_{\varepsilon \to 0} \frac{F(z+\varepsilon) - F(z)}{F(z+\varepsilon) - F(z) + F(y+\varepsilon) - F(y)}$$

Using l'Hôpital's rule,

$$\mu_i(z|M_1, M_2, P_1, P_2) = \lim_{\varepsilon \to 0} \frac{f(z+\varepsilon)}{f(z+\varepsilon) + f(y+\varepsilon)} = \frac{f(z)}{f(z) + f(y)}$$

3 Results

Given that we have already derived consumer demand in the previous section, we start our analysis by studying the pricing decision of firms.

Each firm anticipates the optimal behavior of consumers and chooses price so as to maximize its expected profit, for any given messages of the firms sent at the previous stage. Expression (2.2) for $\widehat{\lambda}$ implies that the profits of firms 1 and 2 are given by

$$\begin{aligned}\pi_1 &= P_1\left(\frac{1}{2}\frac{\frac{P_2-P_1}{t}+var\left(x_2|\mu_2\right)-var\left(x_1|\mu_1\right)}{E\left(x_2|\mu_2\right)-E\left(x_1|\mu_1\right)}+\frac{1}{2}\left(E\left(x_1|\mu_1\right)+E\left(x_2|\mu_2\right)\right)\right)\\ \pi_2 &= P_2\left(1-\frac{1}{2}\frac{\frac{P_2-P_1}{t}+var\left(x_2|\mu_2\right)-var\left(x_1|\mu_1\right)}{E\left(x_2|\mu_2\right)-E\left(x_1|\mu_1\right)}+\frac{1}{2}\left(E\left(x_1|\mu_1\right)+E\left(x_2|\mu_2\right)\right)\right).\end{aligned}$$

Function π_i, $i=1,2$, is a strictly concave, quadratic function of P_i. Hence, as prices do not effect consumers' beliefs about location the profit-maximization problem of each firm is well-defined and the first-order conditions yield the price at which π_i is maximized:[6]

$$\begin{aligned}-\frac{P_1}{t\left(E\left(x_2|\mu_2\right)-E\left(x_1|\mu_1\right)\right)}+\frac{1}{2}\left(\frac{\frac{P_2}{t}+var\left(x_2|\mu_2\right)-var\left(x_1|\mu_1\right)}{E\left(x_2|\mu_2\right)-E\left(x_1|\mu_1\right)}+E\left(x_1|\mu_1\right)+E\left(x_2|\mu_2\right)\right) &= 0\\ 1-\frac{P_2}{t\left(E\left(x_2|\mu_2\right)-E\left(x_1|\mu_1\right)\right)}-\frac{1}{2}\left(\frac{-\frac{P_1}{t}+var\left(x_2|\mu_2\right)-var\left(x_1|\mu_1\right)}{E\left(x_2|\mu_2\right)-E\left(x_1|\mu_1\right)}+E\left(x_1|\mu_1\right)+E\left(x_2|\mu_2\right)\right) &= 0.\end{aligned}$$

The first equation above is the first-order condition for firm 1, $\frac{\partial\pi_1}{\partial P_1}=0$, while the second equation is the first-order condition for firm 2, $\frac{\partial\pi_2}{\partial P_2}=0$. The second equation yields the best-response pricing strategy of firm 2:

$$\begin{aligned}P_2 &= t\left(E\left(x_2|\mu_2\right)-E\left(x_1|\mu_1\right)\right)-\frac{1}{2}\Big(-P_1+t\left(var\left(x_2|\mu_2\right)-var\left(x_1|\mu_1\right)\right)+\\ &\quad +t\left(E^2\left(x_2|\mu_2\right)-E^2\left(x_1|\mu_1\right)\right)\Big)\end{aligned}\tag{3.1}$$

Summing up the two first-order conditions leads to the best-response pricing strategy of firm 1:

$$P_1=2t\left(E\left(x_2|\mu_2\right)-E\left(x_1|\mu_1\right)\right)-P_2\tag{3.2}$$

[6]This derivation makes use of the fact that we are restricting our attention to equilibria where prices do not affect consumers' beliefs regarding location.

Collecting (3.1) and (3.2) results in the solution of the price setting stage of the game:

$$
\begin{aligned}
P_1 &= t\Big(\frac{2}{3}\left(E\left(x_2|\mu_2\right) - E\left(x_1|x_1 \in \Omega_1\right)\right) + \frac{1}{3}\left(E^2\left(x_2|x_2 \in \Omega_2\right) - E^2\left(x_1|\mu_1\right)\right) + \\
&\quad + \frac{1}{3}\left(var\left(x_2|\mu_2\right) - var\left(x_1|\mu_1\right)\right)\Big) \qquad (3.3)\\
P_2 &= t\Big(\frac{4}{3}\left(E\left(x_2|\mu_2\right) - E\left(x_1|\mu_1\right)\right) - \frac{1}{3}\left(E^2\left(x_2|\mu_2\right) - E^2\left(x_1|\mu_1\right)\right) - \\
&\quad - \frac{1}{3}\left(var\left(x_2|\mu_2\right) - var\left(x_1|\mu_1\right)\right)\Big) \qquad (3.4)
\end{aligned}
$$

Plugging expressions (3.3)– (3.4) for prices into the profit functions of the two firms, yields reduced-form profit functions that are expressed in terms of the conditional expectations and variances of x_1 and x_2:

$$
\begin{aligned}
\pi_1 &= \frac{t}{18\left(E\left(x_2|\mu_2\right) - E\left(x_1|\mu_1\right)\right)} \cdot \Big(2\left(E\left(x_2|\mu_2\right) - E\left(x_1|\mu_1\right)\right) + \qquad (3.5)\\
&\quad + \left(E^2\left(x_2|\mu_2\right) - E^2\left(x_1|\mu_1\right)\right) + \left(var\left(x_2|\mu_2\right) - var\left(x_1|\mu_1\right)\right)\Big)^2 \\
\pi_2 &= \frac{t}{18\left(E\left(x_2|\mu_2\right) - E\left(x_1|\mu_1\right)\right)} \cdot \Big(4\left(E\left(x_2|\mu_2\right) - E\left(x_1|\mu_1\right)\right) - \qquad (3.6)\\
&\quad - \left(E^2\left(x_2|\mu_2\right) - E^2\left(x_1|\mu_1\right)\right) - \left(var\left(x_2|\mu_2\right) - var\left(x_1|\mu_1\right)\right)\Big)^2
\end{aligned}
$$

We can now consider stage at which firms decide on the messages they will send. As a special case, consider first the situation where locations of both firms are fully revealed. This means that in all expressions above $E\left(x_i|\mu_i\right) = x_i$, $var\left(x_i|\mu_i\right) = 0$ and profits of firm 1 and 2 are functions of exact locations x_1, x_2. In particular, following the assumption that $x_1 \leq x_2$ (the analogue of $E\left(x_1|\mu_1\right) \leq E\left(x_2|\mu_2\right)$), the profits of firms 1 and 2 are given by:

$$
\begin{aligned}
\pi_1(x_1, x_2) &= \frac{t}{18}(x_2 - x_1)(2 + x_1 + x_2)^2 \qquad (3.7)\\
\pi_2(x_1, x_2) &= \frac{t}{18}(x_2 - x_1)(4 - x_1 - x_2)^2 \qquad (3.8)
\end{aligned}
$$

Both these expressions are strictly positive as long as $x_1 < x_2$. If $x_1 = x_2$, then consumers buy from the firm with the lowest price. The usual Bertrand-type argument then establishes (cf., 3.3 and 3.4) that $P_1 = P_2 = 0$ and so, $\pi_1 = \pi_2 = 0$.

Note that the profit of firm 1 in (3.7) is *decreasing* in x_1, while the profit of firm 2 in (3.8) is *increasing* in x_2. Indeed,

$$
\begin{aligned}
\frac{\partial \pi_1}{\partial x_1} &= (2 + x_1 + x_2)\left(\frac{t}{18}x_2 - \frac{3t}{18}x_1 - \frac{t}{9}\right) < 0 \\
\frac{\partial \pi_2}{\partial x_2} &= (4 - x_1 - x_2)\left(\frac{t}{18}x_1 - \frac{3t}{18}x_2 + \frac{2t}{9}\right) > 0,
\end{aligned}
$$

where the signs of the derivatives are implied by the fact that $0 \leq x_1, x_2 \leq 1$. This finding is consistent with the argument in the standard Hotelling model of location choice. Firms want to be located maximally far from each other as differentiation allows them to charge higher prices, which turns out to outweigh the adverse effect of a decline in demand. This functional dependence of profits on locations plays a key role in the proof of the first theorem:

Theorem 3.1. *There exists a stable belief equilibrium where firms fully disclose their location.*

Theorem 3.1 claims that full disclosure is always an equilibrium of the game. Clearly, the fully revealing equilibrium is not unique since there are many sets of messages with which firms are able to fully disclose their location. In the proof we use strategies where firms disclose their location precisely. This facilitates the proof in the sense that it is impossible for types to imitate each others' message due to the restriction that messages must be truthful. For other fully revealing equilibria, we should verify in addition that this type of imitation is not profitable.

The proof also uses specific out-of-equilibrium beliefs that discourage firms to deviate from their equilibrium strategies. These out-of-equilibrium beliefs are somewhat extreme in the sense that consumers believe that an-out-of-equilibrium message is sent by one of the types with the lowest equilibrium profit of types in the set of types that is consistent with the message, even though all other types that are consistent with the message could also have truthfully send such a message. However, one can show that these out-of-equilibrium beliefs are reasonable in the sense that these extreme out-of-equilibrium beliefs are consistent with the logic of the D1 criterion.[7]

The next result is probably even more important for the general message of the paper than Theorem 3.1 stating the existence of fully disclosing equilibrium. Theorem 3.2 shows that when there is competition between firms, there cannot be a stable belief equilibrium where firms do not perfectly disclose the variety they produce. Thus, even though the fully revealing equilibrium strategies are themselves not unique, the equilibrium outcome of full disclosure is.

Theorem 3.2. *There does not exist a stable belief equilibrium where firms do not fully disclose their location.*

The intuition behind the result of Theorem 3.2 is related to the argument in the standard Hotelling model with location choice, where firms have an incentive to maximally differentiate from each other.

[7]Intuitively, the D1 criterion requires that for a given observed deviation from the equilibrium strategy, consumers believe that such deviation was chosen by the type of a firm that has "most incentives" to deviate. To define which type has "most incentives" to deviate, observe that the profits are completely determined by consumer beliefs about location and not by location itself. Thus, after sending an out-of-equilibrium message, the profits of a deviating firm are independent of its type. Therefore, the incentive to deviate is largest for the type (or types) in $\widehat{M}_i$ with the smallest equilibrium payoff. This way to modify the D1 criterion to this game is suggested by a similar adaptation in Janssen and Roy (2010).

If a firm does not follow a fully revealing strategy, there are locations within its non-fully revealing message that are further away from the perceived location of the rival firm than the own perceived location. These locations have an incentive to deviate by fully disclosing themselves. The reason for this is twofold. First, by fully revealing its location, a firm reduces the uncertainty associated with the location for consumers and given the quadratic transportation costs, any reduction in uncertainty increases the demand *ceteris paribus*. Second, by having a perceived location that is further away from the competitor, a firm can charge higher price and this price effect outperforms the direct effect of a decline in demand.

4 Conclusion

In this paper we developed a duopoly model of horizontal product differentiation. We studied the incentives of a firm to disclose its horizontal product characteristic when this characteristic is known to both firms but not to consumers. Firms first simultaneously choose a message about their location, such that this message is *truthful*, that is, the true location of a firm is consistent with the message. The messages can range from being very precise (indicating the exact location) to very vague. After firms have sent their messages, they simultaneously choose prices. Given the messages and prices, consumers update their beliefs about firms' locations and decide where to buy.

As profits in this environment only depend on expected locations and prices, but not on real locations or types, and as we insist that messages have to contain a grain of truth, we define a stable belief equilibrium where consumers' beliefs concerning a firm's location after observing some out-of-equilibrium message or prices only depend on a firm's own message concerning its location. We argue that this is the natural equilibrium notion in this environment. We find that all stable belief equilibria of the game are such that both firms fully reveal their locations. In other words, there always exists an equilibrium where firms fully disclose their location and there does not exist a stable belief equilibrium where firms do not disclose. This full-disclosure result contrasts with the finding of possible non-disclosure in the literature on horizontal product differentiation in a monopolistic set-up, suggesting that competition plays a key role in determining incentives for firms to disclose.

Intuitively, the reason why in the competitive environment all equilibria must be fully revealing is related to the reason why in the standard Hotelling model with location choice, firms want to maximally differentiate from each other. Suppose that a firm would not fully reveal its location, choosing the same message for different locations. Consumers will then be uncertain about the true location of the firm and hence, will form beliefs such that the extreme locations, sending that particular message, will not obtain full probability mass. These extreme locations have then incentive to deviate to full disclosure for two reasons. On one hand, by fully revealing its location, a firm reduces the uncertainty concerning the location for consumers and with quadratic transportation costs, this increases their demand. On the other hand, by indicating a location that is perceived by consumers as being further away from the competitor, price competition is softened and this price effect outweighs the direct effect of a decline in demand.

In the present model we have considered a simple framework where consumers are uniformly distributed over the unit interval and have quadratic transportation costs. Moreover, disclosure is completely costless and firms know not only their own location, but also the location of their competitors. We have considered this simple framework to focus on the role of competition in providing incentives for firms to fully disclose. In future work, we intend to relax some of these assumptions. For example, the case where firms have purely private information about their product characteristics could be of considerable interest.

Appendix

Proof of Theorem 3.1. Suppose that firms fully reveal their location by truthfully announcing it, i.e., every firm with location x_i sends message $M_i = \{x_i\}$. Since firms cannot lie, the firm of any given type x_i cannot imitate the message of another type. Any deviating message is therefore an out-of-equilibrium message and the proof of an equilibrium then requires to construct a set of out-of-equilibrium beliefs such that given these beliefs, no firm has an incentive to deviate.

Let us consider the following out-of-equilibrium beliefs. For any out-of-equilibrium message $\widehat{M_i}$ sent by firm i, consumers assign probability one to firm i being of type $\widehat{x}(M_i)$ where

$$\widehat{x}(M_i) \in \arg \min_{x_i \in \widehat{M_i}} \pi_i(x_1, x_2)$$

i.e., $\widehat{x}(M_i)$ is any selection from the set of minimizers of the function $\pi_i(x_1, x_2)$ on the set $\widehat{M_i}$.

Observe that given these out-of-equilibrium beliefs, no type of any firm wishes to deviate from the candidate equilibrium strategies. If firm i of type x_i deviates and sends some admissible message $\widehat{M_i} \neq \{x_i\}$, then the subsequent choice of consumers will be as if the true type of firm i is $\widehat{x}(M_i)$ for sure, and since all that matters for the payoff of firm i is her perceived type (and not her true type), the expected continuation payoff after this deviation is exactly equal to $\pi_i(\widehat{x}(M_i), x_j)$. As $x_i \in M_i$ it follows that $\pi_i(\widehat{x}(M_i), x_j) \leq \pi_i(x_i, x_j)$. Therefore, the deviation is not gainful. □

Proof of Theorem 3.2. Suppose that at least one of the two firms does not follow a fully revealing strategy and chooses the same message for different locations. Let types $x_1 \in S_1$ send identical message M_1 and types $x_2 \in S_2$ send identical message M_2, where at least one of the sets S_1, S_2 contains two or more types.[8] Without loss of generality, assume that firm 1 does not fully disclose its location (while firm 2 may disclose or not disclose). In this case, consumers are uncertain about the true location of the non-disclosing firm/firms and form expectations about this location and resulting transportation costs. Again, without loss of generality, we restrict the analysis to the case when $E(x_1|\mu_1) \leq E(x_2|\mu_2)$. If the inequality is strict, the profits of firms 1 and 2 are given by (3.5)–(3.6). If instead $E(x_1|\mu_1) = E(x_2|\mu_2)$, then consider the firm (referred to as firm i) whose equilibrium message has the largest variance (referred to as firm j), that is, $var(x_i|\mu_i) \geq var(x_j|\mu_j)$. The profit of firm i is equal to zero because firm j is at least as attractive to consumers and hence, can either push firm i out of the market by setting $P_j = t(var(x_i|\mu_i) - var(x_j|\mu_j))$ (if $var(x_i|\mu_i) > var(x_j|\mu_j)$) or share the market with firm i but at zero prices (if $var(x_i|\mu_i) = var(x_j|\mu_j)$).

Suppose first that $E(x_1|\mu_1) < E(x_2|\mu_2)$. We prove that if $var(x_1|\mu_1) \geq var(x_2|\mu_2)$, the deviation to full disclosure is profitable for any type y of firm 1 such that $y < E(x_1|\mu_1)$. If the opposite inequality for variances holds, the deviation to full disclosure is profitable for any type z of firm 2 such that $z > E(x_2|\mu_2)$.[9] The proof of this claim relies on the following two observations.[10] First, profit functions π_1 and π_2 are monotonically decreasing in $var(x_1|\mu_1)$ and $var(x_2|\mu_2)$, respectively. This is an immediate implication of (3.5) and (3.6). Second, profit function π_1 in (3.5) is monotonically *decreasing* in $E(x_1|\mu_1)$ when $var(x_1|\mu_1) \geq var(x_2|\mu_2)$, and profit function π_2 in (3.6) is monotonically *increasing* in $E(x_2|\mu_2)$ when the opposite inequality is true, i.e., $var(x_2|\mu_2) > var(x_1|\mu_1)$. To demonstrate this second observation, consider the derivative of π_1 with respect to $E(x_1|\mu_1)$ and the derivative of π_2 with respect to $E(x_2|\mu_2)$ and evaluate their signs. Straightforward calculations lead to

$$\begin{aligned}\frac{\partial \pi_1}{\partial E(x_1|\mu_1)} &= \frac{p_1}{6\left(E(x_2|\mu_2) - E(x_1|\mu_1)\right)^2}\Big(\left(E(x_2|\mu_2) - E(x_1|\mu_1)\right)\left(E(x_2|\mu_2) - 3E(x_1|\mu_1) - 2\right) + \\ &\quad + var(x_2|\mu_2) - var(x_1|\mu_1)\Big) \\ \frac{\partial \pi_2}{\partial E(x_2|\mu_2)} &= \frac{p_2}{6\left(E(x_2|\mu_2) - E(x_1|\mu_1)\right)^2}\Big(\left(E(x_2|\mu_2) - E(x_1|\mu_1)\right)\left(4 - 3E(x_2|\mu_2) + E(x_1|\mu_1)\right) + \\ &\quad + var(x_2|\mu_2) - var(x_1|\mu_1)\Big)\end{aligned}$$

[8] If both sets, S_1 and S_2, contain only one type, then reporting strategies of both firms are fully revealing.

[9] Type y of firm 1 such that $y < E(x_1|\mu_1)$ exists because a) by assumption, firm 1 does not fully disclose its location, so that S_1 is not a singleton, and b) the probability density function $f(x)$ is strictly positive. For the same reason, when $var(x_2|\mu_2) > var(x_1|\mu_1)$, type z of firm 2 such that $z > E(x_2|\mu_2)$ exists.

[10] As the deviation is such that its effect on the variance and the effect on the expected location of consumers both increase profits, we can act as if these two effects can be achieved independently of each other.

Given that $0 \leq E(x_1|\mu_1), E(x_2|\mu_2) \leq 1$, the first expression is strictly *negative* when $var\,(x_1|\mu_1) \geq var\,(x_2|\mu_2)$ and the second expression is strictly *positive* when $var\,(x_2|\mu_2) > var\,(x_1|\mu_1)$.

Now, suppose that $E\,(x_1|\mu_1) = E\,(x_2|\mu_2)$. Then if $var\,(x_1|\mu_1) \geq var\,(x_2|\mu_2)$, the deviation by type y of firm 1 to the fully revealing message is beneficial simply because before the deviation its profit is zero and after the deviation it is positive:

$$\pi_1^D = \frac{t}{18\,(E\,(x_2|\mu_2) - y)} \left(2\,(E\,(x_2|\mu_2) - y) + \left(E^2\,(x_2|\mu_2) - y^2\right) + var\,(x_2|\mu_2)\,\right)^2$$

Similarly, if $var\,(x_2|\mu_2) > var\,(x_1|\mu_1)$, then the deviation by type z of firm 2 to the fully revealing message is beneficial. Thus, at least one firm can always benefit by deviating. Therefore, an equilibrium where firms do not fully disclose their location does not exist. □

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