

Politicians, Redistribution and Intergenerational Conflicts*

Francesco Lancia

Alessia Russo

October 4th, 2013

Abstract

The main objective of this work concerns the study of how political disagreement over the intergenerational redistribution of public resources affects efficiency. When inter-cohort heterogeneity is explicitly considered and human capital accumulation is the main force driving the economic growth, intergenerational conflicts may improve efficiency. We study how political institutions may enhance to implement the efficient allocation. We first characterize in a neoclassical environment the politico-economic equilibrium of a repeated voting game characterized by neither fiscal nor political distortions under credit market constraint. Second, we introduce distortionary taxation on capital, explicitly considering political distortions, which arise when ideologically heterogeneous politicians internalize how political decisions affect their probability of winning elections. We characterize the Markov perfect politico-economic equilibrium in terms of Generalized Euler Conditions, discussing how distortions alter the main finding about the positive correlation between intergenerational political disagreement and efficiency.

JEL Classification: C61, D71, E62, H11

Keywords: capital taxation, intergenerational redistribution, Markovian equilibria, political distortions.

*Francesco Lancia, University of Vienna, Email: francesco.lancia@univie.ac.at. Alessia Russo, University of Oslo, Email: alessia.russo@econ.uio.no. We are indebted to Mark Aguiar, Michele Boldrin, Carlos Garriga, Fernando Martin, and Nicola Pavoni for valuable comments. We also thanks participants at the University of Modena for the useful discussion. We acknowledge the Heinrich Graf Hardegg'sche Stiftung for financial support. All errors are our own.

1 Introduction

Decentralized overlapping generations economies populated by selfish agents typically feature dynamically inefficient asset accumulation paths (Samuelson, 1958). When credit markets are incomplete and human capital is the main engine of growth, two different sources of inefficiencies simultaneously arise. On one hand, young are precluded from borrowing money in order to acquire skills and, in turns, increase future labor productivity. On the other hand, adults are restricted from lending money, having access to a restricted investment portfolio. As a consequence, educational investments tend to be inefficiently low, inducing underaccumulation of human capital, and physical capital tends to overaccumulate, depressing future return to capital. In this scenario, there is room to investigate how intergenerational institutions might improve the welfare of all generations. Previous literature (Boldrin and Montes, 2005; Docquier et al., 2007) has adopted a normative perspective to determine the fiscal scheme, which allows the decentralized economy to reach the efficient allocation.¹

We depart from the previous theoretical contributions by adopting a positive perspective. Specifically, we analyze how voting institutions and intergenerational conflicts over the implementation of redistributive policies affect efficiency and welfare. A workhorse of past literature in political economy (Alesina and Tabellini, 1990; Barro, 1991; Azzimonti, 2011) has studied how political disagreement and uncertainty depress growth and reduce welfare when intra-cohort heterogeneity is explicitly considered. When different groups disagree over the composition of public expenditure and parties compete to detain power via democratic process, governments tend to be endogenously short-sighted. As a result, the economy experiences under-investment of productive assets and, in turns, loss in efficiency.²

Our perspective is opposite to the one stated above. When inter-cohort heterogeneity is explicitly considered and human capital accumulation is the main force driving the economic growth, intergenerational conflicts over the redistribution of public resources may enhance efficiency and improve welfare. Although the preferences of younger agents to sustain productive public spending are growth promoting, nonetheless the public education transfers crowds-out private consumption. As a result, the education spending maximizing welfare turns out to be smaller than that maximizing growth. Consequently, welfare improvements are achievable by reallocating a share of government spending from public education to pork-barrel transfers.

¹Boldrin and Montes (2005) show how an interconnected pension and public education system can replicate the allocation achieved by complete market. The authors formalize public education and PAYG system as two parts of an intergenerational contract where public pension is the return on the investment into the human capital of the next generation. In presence of credit market constraint, the welfare state is justified by the inability of decentralized markets to deliver a Pareto efficient solution. Relaxing the definition of optimality by explicitly considering the positive spillover generated by educational investments, Docquier et al. (2007) show that no justifications to provide public pension benefits emerge, when the dynastic welfare weight is sufficiently high. For realistic values of discount rates, the achievement of efficient allocation is guaranteed by taxing the retirees in order to provide educational subsidy.

²The negative correlation between political instability and private investment has been widely investigated both theoretically (Alesina and Tabellini, 1990) and empirically (Barro, 1991). Azzimonti (2011) extends the previous studies by analyzing a dynamic electoral competition framework. Ruling out commitment devices, the author gives further theoretical support to the role of political stability in mitigating the effects of polarization, dampening the inefficiencies.

This simple idea helps to justify the existence of institutions, which reinforce intergenerational political disagreement and uncertainty as a device for the achievement of self-enforcing politico-economic equilibria closer to the optimal allocation.

To support the positive relation between intergenerational conflicts and efficiency, we present a tractable dynamic politico-economic model in a neoclassical environment. Focusing on a three period OLG economy with endogenous growth generated by human capital accumulation, we determine the subgame Markov perfect equilibrium of a dynamic game characterized by both ideologically heterogeneous voters and office-seeking political parties. Only adult and elderly have voting power. The electoral competition takes place in a majoritarian probabilistic environment, where ideologically heterogeneous parties compete proposing multidimensional fiscal platforms, in order to maximize the probability of winning election.³

Three main sources of distortions characterize the theoretical environment: Incomplete credit markets, physical capital taxation (fiscal distortion), and ideological political competition (political distortion). In order to better perform the analysis and provide clear theoretical insights, we distinguish two scenarios.

The first scenario is characterized by neither fiscal nor political distortions. Consequently, the only source of inefficiency comes from the credit market constraint. In equilibrium the presence of swing voters determines the emergence of intergenerational conflicts over the redistribution of public resources. The ratio of the idiosyncratic ideological densities between the two cohorts of voters represents a simple quantitative measurement of the political disagreement. The median voter framework, where adults are the decisive voter, can be considered as an extreme benchmark case with no intergenerational political disagreement. In equilibrium the politicians implement a multidimensional platform characterized by: i) negative transfers from the elderly, ii) positive transfers to the adults in order to subsidize consumption, and iii) high investment in public education. Differently from the previous literature (Azariadis and Galasso, 2002; Forni, 2005), which studies how intergenerational transfers might be sustained in a median voter framework with exogenous growth, in our environment the relevant state variable is the total return to capital, which is affected by both human and physical capital. Adults have incentives to invest in the education of young in order to accumulate human capital and, in turns, increase the future return to capital. At the same time the incentives to transfer public resource backward (i.e. pension benefits in the form of PAYGO system) do not arise. Adults perfectly anticipate that when old they are prevented from grabbing public resources by exerting political power. As a consequence, in equilibrium public education investment is financed by taxes paid by the elderly. Because of the resulting overaccumulation of both physical and human capital the economy turns out to be still characterized by dynamic inefficiency. In presence of ideological uncertainty, as soon as swing voters on behalf of the elderly emerge, the equilibrium political platform reverses, turning out to be characterized by: i) positive transfers to the elderly, ii) negative transfers from the adults, and iii) lower invest-

³According to Persson and Tabellini (2000), we adopt an opportunistic models of electoral competition, where politicians extract rent from being in power in order to maximize the probability of winning elections. While this approach dates back to the 1970s, its resurgence in popularity stems from Lindbeck and Weibull (1987).

ment in public education. The active participation of the elderly to the political debate induces the emergence of political disagreement on the redistribution of public resources that, in turns, improves the overall economic efficiency. The elderly claim for positive transfers induces a simultaneous reduction in both physical and human capital accumulation, partially offsetting the dynamic inefficiency.⁴

The second scenario concerns the analyses of the politico-economic equilibrium when both fiscal and political distortions are explicitly introduced. Public expenditures are financed through both labor and capital income tax. In absence of commitment technology, the distortionary taxation adds an additional source of inefficiency with respect to the previous analyses.⁵ Furthermore, forward-looking and ideologically heterogeneous politicians might strategically manipulate their probability of winning future election through the current fiscal platform.⁶ We characterize the time-consistent Markov perfect politico-economic equilibrium in terms of Generalized Euler Conditions (GEEs). Finally, we discuss how fiscal and political distortions alter the main finding about the positive correlation between intergenerational political disagreement and efficiency.

The remainder of the paper is organized as follows. Section 2 presents the environment. Section 3 characterizes the first best allocation. Section 4 describes the politico-economic equilibrium in the perfect forward-looking scenario. Section 5 fully characterizes the case with neither fiscal nor political distortions, providing a closed-form economy example. Section 6 shows how distortionary taxation and ideological competition may affect the equilibrium prescriptions, respectively. Section 7 concludes. The appendix contains all the proofs.

2 The Model

Consider an OLG economy populated by an infinite number of ideological heterogeneous agents, living up to three-periods: young, adult and old. We denote by $\tau \in \{1, 2\}$ the adult and elderly cohorts, respectively. Time is discrete, indexed by t , and runs from zero to infinity. Population growth rate is exogenous and equal to zero with a unitary mass for each cohort. Furthermore, there are two ideological heterogeneous infinite living parties, left and right, denoted by $i \in \{\mathcal{L}, \mathcal{R}\}$, who compete proposing at each time an electoral platform in order to maximize their probability of winning election.

⁴The equilibrium outcome is opposite to the one determined by Song et al. (2011). They study how the political participation of young agents may discipline fiscal policies in the presence of public debt and public good provision. Contrarily, we find that the political participation of elderly support fiscal discipline when human capital is the main engine of growth and intergenerational transfers are considered.

⁵This component is similar to the optimality conditions derived by Klein, Krusell and Rios-Rull (2008).

⁶Azzimonti (2011) studies how political competition may endogenously affect the probability of winning future elections. However, considering a partisan model of electoral competition and limiting the analyses to symmetric strategy, the author finds that in equilibrium the probability of being elected are constant and only affected by the exogenously given incumbency power index. As a consequence, it turns out to be no strategically manipulated by politicians.

2.1 Household

The intertemporal random preference of a representative agent j born at time $t - 1$ and living at time t is defined as follows:

$$u(c_{it}^1) + \sigma_{jit}^1 + \beta E^{\mathcal{P}_{it}} \left(u(c_{it+1}^2) + \sigma_{jit+1}^2 \right) \quad (1)$$

where $\beta \in (0, 1)$ is the individual discount factor and $i_t \in \{\mathcal{L}_t, \mathfrak{R}_t\}$. \mathcal{P}_{it} denotes the endogenous probability of party \mathfrak{R} of being in power at time $t + 1$ when the incumbent party is i_t . σ_{jit}^τ is the individual ideological bias toward party \mathfrak{R} of agent j belonging to cohort τ at time t . c_{it}^1 represents the consumption when adult, and c_{it+1}^2 denotes the consumption when old.⁷ Young do not consume.

The stochastic component of preferences can be decomposed into two terms, as follows:

$$\sigma_{jit}^\tau = (\theta_t + \psi_{jt}^\tau) D_{it}$$

where D_{it} is an indicator function, which is equal to 1 if \mathfrak{R}_t is in power at time t , or zero otherwise.

Assumption 1 (Ideology) *The random variables $\psi_{jt}^\tau \sim \left[-\frac{1}{2\psi^\tau}, \frac{1}{2\psi^\tau}\right]$ and $\theta_t \sim \left[-\frac{1}{2\theta}, \frac{1}{2\theta}\right]$ are i.i.d., uniformly distributed and centered over zero.*

ψ_{jt}^τ represents the idiosyncratic shock, whose distribution is cohort specific, and measures voter j 's individual preferences toward party \mathfrak{R} . θ_t represents an aggregate shock and measures the average relative popularity of candidates from party \mathfrak{R} relative to those from party \mathcal{L} .

Assumption 2 (Utility) *The function $u : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is twice continuously differentiable concave function with $\lim_{c \rightarrow 0} u_c(c) = \infty$.*

When young, agents spend all their time endowment in acquiring skills if the *forward productive transfers*, f_{it} , is publicly provided without having access to private credit market. When adult, individuals supply inelastically labor, pay taxes, consume and save for retirement. When old, agents pay taxes and consume their entire income, which is composed of capitalized saving and the publicly provided *backward pork-barrel transfers*, b_{it} . The total income tax rate is equal to π_{it} . The individual budget constraints for adults and old are, respectively:

$$c_{it}^1 \leq (1 - \pi_{it}) w_t h_t - s_t \quad (2)$$

$$c_{it+1}^2 \leq (1 - \pi_{it+1}) R_{t+1} s_t + b_{it+1} \quad (3)$$

At the initial time, $t = 0$, there is an exogenous human capital endowment, $h_0 > 0$. The budget constraint of each adult is then equal to $c_{i_0}^1 = (1 - \pi_{i_0}) w_0 h_0 - s_0$. At the same the

⁷Apart from the different time horizon, the unique source of heterogeneity among agents is the ideology. Given that the ideological component is independent from the perceived benefits of consumption, saving decisions are identical across agents. Ideological heterogeneity only affects the individual voting decisions.

elderly are endowed with an exogenously given physical capital, $k_0 > 0$. Their individual budget constraint is equal to $c_{i_0}^2 = (1 - \pi_{i_0}) R_0 k_0 + b_{i_0}$.

2.2 Technology

At each time t the economy produces a single consumption good, y_t , combining physical, k_t , and human capital, h_t , according to a constant return to scale technology, $y_t = \Theta(k_t, h_t)$.

Assumption 3 (Production Technology) *The function $\Theta : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ is strictly monotonic increasing and strictly concave with $\Theta(0, h_t), \Theta(k_t, 0) \geq 0$ and $\Theta_{kh}, \Theta_{hk} \geq 0$.*

Let us denote with $\tilde{y}_t \equiv \frac{y_t}{h_t}$ and $\tilde{k}_t = \frac{k_t}{h_t}$ the per-efficiency units of final good production and physical capital, respectively. Physical capital fully depreciates each period. Under Assumption 4 it follows that $y_t = h_t \Theta\left(\frac{k_t}{h_t}, 1\right) \equiv h_t \vartheta(\tilde{k}_t)$ and, in turns, $\tilde{y}_t = \vartheta(\tilde{k}_t)$. As a consequence, the inverse demands for factor prices are $\Theta_k = \vartheta_{\tilde{k}}(\tilde{k}_t)$ and $\Theta_h = \vartheta(\tilde{k}_t) - \tilde{k}_t \vartheta_{\tilde{k}}(\tilde{k}_t)$, with $\Theta_{hk} = \Theta_{kh} = \frac{\vartheta_{\tilde{k}}(\tilde{k}_t)}{h_t} \frac{\vartheta(\tilde{k}_t) - \tilde{k}_t \vartheta_{\tilde{k}}(\tilde{k}_t)}{\vartheta(\tilde{k}_t)}$ and $\Theta_{kk} = -\frac{\vartheta_{\tilde{k}}(\tilde{k}_t)}{k_t} \frac{\vartheta(\tilde{k}_t) - \tilde{k}_t \vartheta_{\tilde{k}}(\tilde{k}_t)}{\vartheta(\tilde{k}_t)}$.

The human capital is produced according to a constant return to scale technology, which combines parental education and public investment as complement factors:

$$h_{t+1} = H(h_t, f_{it}) \quad (4)$$

Assumption 4 (Human Capital Technology) *The function $H : \mathbb{R}_+^2 \rightarrow \mathbb{R}_+$ is strictly monotonic increasing and strictly concave with $H(0, f_{it}), H(h_t, 0) \geq 0$ and $H_{hf}, H_{fh} \geq 0$.*

Let us denote with $\tilde{f}_{it} \equiv \frac{f_{it}}{h_t}$ the per-efficient units of productive transfers. Under Assumption 4 it follows that $h_{t+1} = h_t H\left(1, \frac{f_{it}}{h_t}\right) \equiv h_t \varphi(\tilde{f}_{it})$. As a consequence, the human capital growth rate is equal to $\frac{h_{t+1}}{h_t} = \varphi(\tilde{f}_{it})$, which represents also the economy's growth rate. The marginal impact of parental education and forward productive transfers on the human capital production are $H_h = \varphi(\tilde{f}_{it}) - \tilde{f}_{it} \varphi_{\tilde{f}}(\tilde{f}_{it})$ and $H_f = \varphi_{\tilde{f}}(\tilde{f}_{it})$, respectively.

2.3 Fiscal Constitution

In order to provide the intergenerational transfers, agents have to devise a politician. In each period, the politician raises revenues through income taxes and uses the proceeds to purchase consumption good to be converted into intergenerational transfers. The politicians cannot use lump-sum taxes. They can only levy a proportional tax on labor and capital income. The tax rates on the two sources of income are equal. We assume for simplicity that the politician is prevented from borrowing: The public balance must hold in every period. This implies that in each period total benefits paid to the agents equalize total contributions collected from tax payers. Then for each elected politician belonging to party i_t , the balanced budget constraint condition can be written as:

$$\pi_{i_t} y_t = f_{i_t} + b_{i_t} \quad (5)$$

Eq. (5) allows us to reduce the multidimensionality of political platform, $z_{i_t} \equiv \{f_{i_t}, b_{i_t}, \pi(f_{i_t}, b_{i_t})\}$.

Assumption 5 (Feasibility) *At each time t and for each party i_t :*

- i) consumption of elderly agent is non negative, i.e. $b_{i_t} > -(1 - \pi_{i_t}) R_t k_t$;*
- ii) forward productive transfers are non negative, i.e. $f_{i_t} \geq 0$;*
- iii) income tax rate is a percentage of total production, i.e. $\pi_{i_t} \leq 1$.*

3 First Best Allocation

In this section we characterize the efficient allocation chosen by a benevolent planner with a commitment technology in the absence of distortionary taxation. Lump sum taxes are used to finance forward productive transfer and backward pork-barrel transfers. The benevolent planner takes the initial level of human and physical capital $\{h_0, k_0\}$ as given, and chooses a sequence $\{c_t^1, c_t^2, f_t, h_{t+1}, k_{t+1}\}_{t=0}^{\infty}$ in order to maximize a weighted sum of lifetime utilities over generations. The welfare weight of each representative dynasty is exogenously given, $\delta \in (0, 1)$. The corresponding maximization problem is equal to:

$$\max_{\{c_t^1, c_{t+1}^2, f_t, h_{t+1}, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta^{t+1} (u(c_t^1) + \beta u(c_{t+1}^2)) + \beta u(c_0^2)$$

subject to the aggregate resource constraint and the human capital technology:

$$\begin{aligned} c_t^1 + c_t^2 + k_{t+1} + f_t - h_t \vartheta(\tilde{k}_t) &\leq 0, \forall t \quad (\mu_t \delta^{t+1}) \\ h_{t+1} - h_t \varphi(\tilde{f}_t) &\leq 0, \forall t \quad (\eta_t \delta^{t+1}) \end{aligned}$$

where $(\mu_t \delta^{t+1})$ and $(\eta_t \delta^{t+1})$ are the associated Lagrange multipliers. Removing the functional arguments, the first order conditions of the Lagrangian turn out to be equal to:

$$\begin{aligned} c_t^1 &: u_{c_t^1} = \mu_t \\ c_{t+1}^2 &: \beta u_{c_{t+1}^2} = \delta \mu_{t+1} \\ f_t &: \mu_t = \eta_t \varphi \tilde{f}_t \\ h_{t+1} &: \eta_t = \delta \mu_{t+1} \left(\vartheta(\tilde{k}_{t+1}) - \tilde{k}_{t+1} \vartheta_{\tilde{k}}(\tilde{k}_{t+1}) \right) + \delta \eta_{t+1} \left(\varphi_{t+1} - \tilde{f}_{t+1} \varphi_{\tilde{f}_{t+1}} \right) \\ k_{t+1} &: \mu_t = \delta \mu_{t+1} \vartheta_{\tilde{k}}(\tilde{k}_{t+1}) \end{aligned}$$

together with the transversality conditions (TVCs):

$$\lim_{t \rightarrow \infty} \delta^{t+1} \mu_t k_{t+1} = 0 \tag{6}$$

$$\lim_{t \rightarrow \infty} \delta^{t+1} \eta_t h_{t+1} = 0 \tag{7}$$

Rearranging the first order conditions, the following Euler conditions, or first order wedges, for the optimal allocations must be satisfied:

$$\Delta^r \equiv \beta u_{c_t^2} - \delta u_{c_t^1} = 0 \quad (8)$$

$$\Delta^s \equiv u_{c_t^1} - \beta \vartheta_{\tilde{k}}(\tilde{k}_{t+1}) u_{c_{t+1}^2} = 0 \quad (9)$$

$$\Delta^f \equiv \varphi_{\tilde{f}_t} - \frac{\vartheta_{\tilde{k}}(\tilde{k}_{t+1})}{\vartheta(\tilde{k}_{t+1}) - \tilde{k}_{t+1} \vartheta_{\tilde{k}}(\tilde{k}_{t+1})} (1 - \varepsilon_{t+1}) = 0 \quad (10)$$

where $\varepsilon_{t+1} \equiv \frac{\varphi_{\tilde{f}_t}}{\vartheta_{\tilde{k}}(\tilde{k}_{t+1})} \frac{\varphi_{t+1} - \tilde{f}_{t+1} \varphi_{\tilde{f}_{t+1}}}{\varphi_{\tilde{f}_{t+1}}}$ denotes the education spillover expressed as a fraction of the education cost. It fully describes the impact of current education investment on future level of human capital through the channel of parental investment. The first condition, Eq. (8), captures the redistribution wedge between current adult and old cohorts. The second condition, Eq. (9), describes the optimal saving choice, determining the optimal accumulation of physical capital. The planner chooses k_{t+1} in order to equate the marginal cost, in terms of foregone consumption, to the discounted marginal benefits of savings. The third condition, Eq. (10), reflects the direct effect of productive transfers on the utility of the adults in terms of current cost and expected benefits, which yield the optimal accumulation of human capital. Suppose $\varepsilon_{t+1} = 0$, then Eq. (9) and (10) are equivalent to the necessary conditions of a competitive equilibrium with no credit market constraints, where young can borrow money at the market interest rate.

Definition 1 (Pareto Efficient Allocation) *For any initial conditions $\{h_0, k_0\}$ the optimal allocation $\{c_t^{1*}, c_t^{2*}, f_t^*, h_{t+1}^*, k_{t+1}^*\}_{t=0}^{\infty}$ satisfies the conditions (6), (7), (8), (9) and (10) for all $t \geq 0$.*

4 Politico-Economic Equilibrium

This section characterizes the politico-economic equilibrium of a dynamic game played among generations in a repeated voting setting. We consider a majoritarian probabilistic voting framework where office-seeking ideologically heterogeneous parties compete in order to maximize their probability of winning election, internalizing the impact of the proposed fiscal platform on the outcome of future elections.

As in Krusell et al. (1997), in order to determine a time consistent solution of the game we solve for the Markov subgame perfect equilibrium using backward procedure, ruling out the assumption of commitment. Specifically, we focus on a "fundamental" equilibrium which is a limit of finite-horizon equilibria, by restricting to continuously differentiable policy functions.

4.1 Timing

To fully describe the main mechanism underlying the resolution strategy of the politico-economic equilibrium, let us provide a complete description of the timing of the game:

- i. at the beginning of each time and before the realization of shocks, the two parties compete proposing a political platform (Political Competition Stage);
- ii. the ideological shocks are realized, the election takes place and the party, which casts the majority of votes, wins election (Electoral Stage);
- iii. the outcome of the political competition is realized and the promised political platform is implemented;
- iv. agents take economic decisions of saving and firms produce (Competitive Equilibrium given Policies).

4.2 Competitive Equilibrium given Fiscal Policies

In a competitive equilibrium, adults choose their lifetime consumption taking factor prices, fiscal policies and probabilities of winning elections as given. Maximizing Eq. (1) subject to the individual budget constraints (2) and (3) and feasibility constraints $c_{i_t}^1 > 0$ and $c_{i_{t+1}}^2 > 0$, the following first order conditions for interior solutions must hold:

$$0 = u_c(c_{i_t}^1) - \beta E^{\mathcal{P}_{i_t}} \left[(1 - \pi_{i_{t+1}}) R_{t+1} u_c(c_{i_{t+1}}^2) \right] \quad (11)$$

In equilibrium by implicit function theorem a unique non-negative saving function exists, i.e. $s_t = K(I_{i_t}^1, I_{i_{t+1}}^2)$, where $I_{i_t}^1 \equiv (1 - \pi_{i_t}) w_t h_t$ and $I_{i_{t+1}}^2 \equiv \mathcal{P}_{i_t} \frac{b_{\mathfrak{R}_{t+1}}}{(1 - \pi_{\mathfrak{R}_{t+1}}) R_{t+1}} + (1 - \mathcal{P}_{i_t}) \frac{b_{\mathcal{L}_{t+1}}}{(1 - \pi_{\mathcal{L}_{t+1}}) R_{t+1}}$. Then the capital market clears when:

$$k_{t+1} = K(I_{i_t}^1, I_{i_{t+1}}^2) \quad (12)$$

Firms produce in a perfectly competitive environment, then in equilibrium they choose the level of employment of capital and the effective units of labor so as to maximize profits, i.e. $\max_{\{k_t, h_t\}} [\Theta(k_t, h_t) - w_t h_t - R_t k_t]$. Firms optimality and markets clearing imply that factor prices are given by the marginal productivity of each factor:

$$R_t = \Theta_{kt} \quad (13)$$

$$w_t = \Theta_{ht} \quad (14)$$

where $\Theta_{kt} \equiv \Theta_k(k_t, h_t)$ and $\Theta_{ht} \equiv \Theta_h(k_t, h_t)$.

Definition 2 (Intertemporal Competitive Equilibrium) *Given the initial conditions $\{h_0, k_0\}$, the sequence of policies $\{f_{i_t}, b_{i_t}\}_{t=0}^{\infty}$ and the probability of winning elections $\{\mathcal{P}_{i_t}\}_{t=0}^{\infty}$ for each possible elected party i_t , an intertemporal competitive equilibrium with perfect foresight is a sequence of allocations and factor prices $\{c_{i_t}^1, c_{i_t}^2, R_t, w_t, h_{t+1}, k_{t+1}\}_{t=0}^{\infty}$ such that, for all $t \geq 0$:*

- i) the allocation solves the maximization problem of adult household, i.e. Eq. (11) is satisfied;

iii) the factor prices are consistent with the profit maximization of firms, i.e. Eq. (13) and (14) are satisfied;

iv) the private good market clears:

$$c_{i_t}^1 + c_{i_t}^2 + k_{t+1} + f_{i_t} = \Theta(k_t, h_t) \quad (15)$$

v) the markets for production inputs clear in period t , i.e. Eq. (4) and (12) hold.

After plugging Eq. (12) into Eq. (2) and (3) and using the Eq. (5) the following individual consumption levels are attained:

$$c_{i_t}^1 = \mathcal{C}_{i_t}^1(\pi_{i_t}, h_t, k_t, k_{t+1}) \equiv I_{i_t}^1 - k_{t+1} \quad (16)$$

$$c_{i_{t+1}}^2 = \mathcal{C}_{i_{t+1}}^2(\pi_{i_{t+1}}, b_{i_{t+1}}, h_{t+1}, k_{t+1}) \equiv (1 - \pi_{i_{t+1}}) \Theta_{k_{t+1}} k_{t+1} + b_{i_{t+1}} \quad (17)$$

Let us denote with $u(\mathcal{C}_{i_t}^1(\pi_{i_t}, h_t, k_t, k_{t+1}))$ and $u(\mathcal{C}_{i_t}^2(\pi_{i_t}, b_{i_t}, h_t, k_t))$ the current utility that adults and old receive in equilibrium, respectively.

4.3 Electoral Stage

After the political proposal, $\{f_{i_t}, b_{i_t}\}$, and the realization of the ideological shocks, agents vote. Let us denote by $V^\tau(k_t, h_t)$ the indirect utility of agents belonging to cohort τ when party \mathcal{L}_t wins the election, whereas by $W^\tau(k_t, h_t)$ the indirect benefits achieved by τ -cohort agents when party \mathfrak{R}_t is in power, net of the ideological shock. The indirect utility of elderly is just characterized by their current utility. Contrarily, the adults have to internalize also the next-period ideological shock. Consequently, for the old-age voter j the voting decision is based on:

$$\max \{V^2(k_t, h_t), W^2(k_t, h_t) + \theta_t + \psi_{jt}^2\}$$

By using Eq. (17) the individual indirect utility for elderly are give by:

$$V^2(k_t, h_t) = u(\mathcal{C}_{\mathcal{L}_t}^2) \quad \text{and} \quad W^2(k_t, h_t) = u(\mathcal{C}_{\mathfrak{R}_t}^2) \quad (18)$$

Equivalently, for the adult-age voter j the voting decision is based on:

$$\max \{V^1(k_t, h_t), W^1(k_t, h_t) + \theta_t + \psi_{jt}^1\}$$

By using Eq. (16) and (17), the individual indirect utilities for adults are equal to:

$$V^1(k_t, h_t) = u(\mathcal{C}_{\mathcal{L}_t}^1) + \beta \mathcal{V}^2(k_{t+1}, h_{t+1}; \mathcal{L}_t) \quad (19)$$

$$W^1(k_t, h_t) = u(\mathcal{C}_{\mathfrak{R}_t}^1) + \beta \mathcal{W}^2(k_{t+1}, h_{t+1}; \mathfrak{R}_t) \quad (20)$$

where the adults' continuation values are:

$$\mathcal{V}^2(k_{t+1}, h_{t+1}; \mathcal{L}_t) \equiv E(\mathcal{P}_{\mathcal{L}_t} (W_{\mathcal{L}_t}^2 + \theta_{t+1} + \psi_{jt+1}^2) + (1 - \mathcal{P}_{\mathcal{L}_t}) V_{\mathcal{L}_t}^2)$$

$$\mathcal{W}^2(k_{t+1}, h_{t+1}; \mathfrak{R}_t) \equiv E(\mathcal{P}_{\mathfrak{R}_t} (W_{\mathfrak{R}_t}^2 + \theta_{t+1} + \psi_{jt+1}^2) + (1 - \mathcal{P}_{\mathfrak{R}_t}) V_{\mathfrak{R}_t}^2)$$

with $W_{i_t}^2 \equiv W^2(k_{t+1}, h_{t+1}; i_t)$ and $V_{i_t}^2 \equiv V^2(k_{t+1}, h_{t+1}; i_t)$ are the indirect utilities when old, conditional on the previous period incumbent party, i_t .⁸

Consequently, the elderly and the adults decide to vote for party \mathfrak{R}_t as long as:

$$\psi_{jt}^2 \geq \psi^2(k_t, h_t) \equiv V^2(k_t, h_t) - W^2(k_t, h_t) - \theta_t \quad (21)$$

$$\psi_{jt}^1 \geq \psi^1(k_t, h_t) \equiv V^1(k_t, h_t) - W^1(k_t, h_t) - \theta_t \quad (22)$$

Formally, $\psi^2(k_t, h_t)$ and $\psi^1(k_t, h_t)$ represent the swing voter in cohort 2 and 1, respectively. Let us denote by $\Delta u(\mathcal{C}_t^\tau) \equiv u(\mathcal{C}_{\mathcal{L}_t}^\tau) - u(\mathcal{C}_{\mathfrak{R}_t}^\tau)$ the difference in utility generated by the implementation of alternative fiscal platforms. Using Eq. (18), (19), (20), (21), and (22), the share of elderly voters for party \mathfrak{R} is equal to:

$$m_t^2 \equiv \frac{1}{2} - \psi^2(\Delta u(\mathcal{C}_t^2) - \theta_t)$$

whereas the adults' share is equal to:

$$m_t^1 \equiv \frac{1}{2} - \psi^1((\Delta u(\mathcal{C}_t^1) - \theta_t) + \beta(\mathcal{V}^2(k_{t+1}, h_{t+1}; \mathcal{L}_t) - \mathcal{W}^2(k_{t+1}, h_{t+1}; \mathfrak{R}_t)))$$

Under majoritarian rule, party \mathfrak{R}_t wins the election if $m_t \equiv m_t^1 + m_t^2 > 1$. Consequently, θ_t must be larger than a certain threshold level equal to:

$$\begin{aligned} \bar{\theta}(k_t, h_t) &\equiv \frac{1}{\psi^1 + \psi^2} \left(\frac{\psi^2}{\psi^1} \Delta u(\mathcal{C}_t^2) + \Delta u(\mathcal{C}_t^1) \right) \\ &+ \frac{1}{\psi^1 + \psi^2} \beta (\mathcal{V}^2(k_{t+1}, h_{t+1}; \mathcal{L}_t) - \mathcal{W}^2(k_{t+1}, h_{t+1}; \mathfrak{R}_t)) \end{aligned} \quad (23)$$

Let $\bar{\theta}_t \equiv \bar{\theta}(k_t, h_t)$ and denote with \mathcal{P}_{i_t} the endogenous probability that party \mathfrak{R} wins the election if the incumbent party is i_t , as follows:

$$\mathcal{P}_{i_t} \equiv \Pr(\theta_{t+1} > \bar{\theta}_{t+1}) = \frac{1}{2} - \theta \bar{\theta}(k_{t+1}, h_{t+1}) \quad (24)$$

and equivalently for party \mathcal{L} :

$$1 - \mathcal{P}_{i_t} \equiv \Pr(\theta_{t+1} < \bar{\theta}_{t+1}) = \frac{1}{2} + \theta \bar{\theta}(k_{t+1}, h_{t+1}) \quad (25)$$

⁸From the adults' perspective the internalization of the t -period incumbent is relevant for their voting decisions. Suppose party i_t wins election at time t , then it will implement policies, which will affect the relevant state variables of the economy. Consequently, both the probability of being re-elected and the future implemented policies will turn out to be conditioned by the identity of the current incumbent party.

The incumbent party i_t affects the probability of re-election through the impact of fiscal policies on the economy's relevant state variables.

4.4 Political Competition Stage

Let us turn to the Political Competition Step where parties have to determine the equilibrium political platform. Specifically, the equilibrium objective functions we are interested in are: The forward and backward transfer policies as a function of the relevant payoff state variables of the economy, i.e. human and physical capital, namely $\mathcal{F}_{i_t}(k_t, h_t)$ and $\mathcal{B}_{i_t}(k_t, h_t)$, the probability of \mathfrak{R} to win the election when the incumbent party is i_t , $\mathcal{P}_{i_t}(k_{t+1}, h_{t+1})$, and, finally, the rules governing the evolution of the payoff-relevant state variables, $k_{t+1} = \mathcal{K}(f_{i_t}, b_{i_t}, k_t, h_t)$ and $h_{t+1} = H(f_{i_t}, h_t)$. The parties' objective function concerns both the maximization of the probability of winning elections and the minimization of the distance from their ideological position:

$$\max_{z_{\mathfrak{R}_t}} -l(z_{\mathfrak{R}_t} - \bar{z}_{\mathfrak{R}}) + \left(\frac{1}{2} - \theta\bar{\theta}_t\right) \quad (26)$$

$$\max_{z_{\mathcal{L}_t}} -l(z_{\mathcal{L}_t} - \bar{z}_{\mathcal{L}}) + \left(\frac{1}{2} + \theta\bar{\theta}_t\right) \quad (27)$$

where $l(z_{i_t} - \bar{z}_i)$ is a loss function measuring the distance of the proposed platform, z_{i_t} , from the parties' ideological bliss points, \bar{z}_i , with $l(0) = 0$.⁹ When political competition is not ideological, parties simply disregard the first part of Eq. (26) and (27).

Definition 3 (Markov Politico-Economic Equilibrium) *A Markov perfect politico-economic equilibrium is an intertemporal competitive equilibrium, $\{c_{i_t}^1, c_{i_t}^2, R_t, w_t, h_{t+1}, k_{t+1}\}_{t=0}^{\infty}$, a sequence of policies, $\{f_{i_t}, b_{i_t}\}_{t=0}^{\infty}$, and probabilities of being elected, $\{\mathcal{P}_{i_t}\}_{t=0}^{\infty}$, such that:*

- i) $f_{i_t} = \mathcal{F}_{i_t}(k_t, h_t)$, $b_{i_t} = \mathcal{B}_{i_t}(k_t, h_t)$ and Eq. (5) holds at each time t ;
- ii) \mathcal{P}_{i_t} is determined by Eq. (24) and (25);
- iii) party \mathfrak{R}_t and \mathcal{L}_t maximize Eq. (26) and (27), respectively, subject to the constraints (4) and (12).

After plugging the equilibrium policy rules, $\mathcal{F}_{i_t}(\cdot)$ and $\mathcal{B}_{i_t}(\cdot)$, and the conditional probability of re-election, $\mathcal{P}_{i_t}(\cdot)$, into Eq. (12) we obtain:

$$k_{t+1} = K(f_{i_t}, b_{i_t}, h_t, k_t, \Theta_{kt+1}, \mathcal{P}_{i_t}(\cdot), \mathcal{F}_{i_{t+1}}(\cdot), \mathcal{B}_{i_{t+1}}(\cdot)) \quad (28)$$

By using Eq. (4) and rearranging the terms, we rewrite Eq. (28) as follows:

$$k_{t+1} = \mathcal{K}(f_{i_t}, b_{i_t}, h_t, k_t)$$

⁹The loss function $l(z_{i_t} - \bar{z}_i)$ can be interpreted as the reputational cost parties sustain if they implement policies which are far from their ideological position.

where \mathcal{K} fully describes the evolution of private capital under the one-period deviation. Since households take policies as given, due to the sequential timing of the game, Eq. (11) becomes as follows:

$$u_c(C_{i_t}^1) - \beta E^{P_{i_t}} \left[(\Theta_{t+1} - \mathcal{F}_{i_{t+1}} - \mathcal{B}_{i_{t+1}}) \frac{\Theta_{kt+1}}{\Theta_{t+1}} u_c(C_{i_{t+1}}^2) \right] = 0$$

We consider two different scenarios. The first is characterized by neither fiscal nor political distortions. Specifically, we assume taxes are levied only on labor income and parties do not take care of their own ideological position. In the second case taxes are levied on both labor income and capital return, thus generating fiscal distortions. Furthermore, we introduce political distortions through parties' ideological competition.

5 No Distortionary Case

Let us first consider the case with neither fiscal nor political distortions. As a consequence, public expenditure are financed through labor income tax and parties don't take into account their ideological position. We denote by $\phi \equiv \frac{\psi^2}{\psi^1}$ the ratio of the idiosyncratic ideological densities between the two voters' cohorts.

Proposition 1 *A unique Markov perfect politico-economic equilibrium, characterized by parties \mathcal{L}_t and \mathcal{R}_t proposing the same political platform, $z_{\mathcal{L}_t} = z_{\mathcal{R}_t} = z_t$, exists. Let $f_{t+1} = \mathcal{F}(k_{t+1}, h_{t+1})$ and $b_{t+1} = \mathcal{B}(k_{t+1}, h_{t+1})$ be the equilibrium policy rules, the parties maximization program is:*

$$\begin{aligned} \max_{\{b_t, f_t, h_{t+1}, k_{t+1}\}} & \phi u(C^2(b_t, h_t, k_t)) + u(C^1(f_t, b_t, h_t, k_t, k_{t+1})) \\ & + \beta u(C^2(\mathcal{B}_{t+1}, h_{t+1}, k_{t+1})) \end{aligned}$$

under the constraint:

$$u_c(C_t^1) - \beta \Theta_{kt+1} u_c(C_{t+1}^2) = 0 \tag{29}$$

Proof. (See appendix). ■

Since the two parties aim to simply maximize their probability of winning election, in equilibrium they propose the same fiscal platform. Specifically, they simply maximize a convex combination of the utility of the current living voters, where the weights reflect the sensitivity of voting behavior to policy changes. Consequently, in equilibrium the probability of winning elections conditioned on the previous-period incumbent i_t is constant over time and equal to $\mathcal{P}_{i_t} = \frac{1}{2}$. It turns out that parties cannot endogenously manipulate the probability of winning future elections through their current policy platform proposal.

The parameter ϕ acquires a crucial theoretical interpretation. It is like a measure of the intergenerational political disagreement over the redistribution of public resources. If $\phi = 0$, given the unitary mass for each cohort, the adults are the median voter and political decisions are taken in order to maximize their intertemporal utility, without considering the participation constraints of both young and elderly. As soon as $\phi > 0$, due to the participation constraint

of the elderly, a conflict over the equilibrium redistribution of public resources emerges. In this section we explore how intergenerational political conflicts, i.e. $\phi \neq 0$, may improve the overall economic efficiency in an economy characterized by productive human capital and storage technologies.

By applying envelope theorem and using Eq. (16), (17), and (29), the Euler conditions with respect to the policies rules f_t and b_t are equal to, respectively:¹⁰

$$0 = -1 + \frac{1}{\Theta_{kt+1}} \left(\frac{\partial \mathcal{B}_{t+1}}{\partial f_t} + k_{t+1} \frac{\partial \Theta_{kt+1}}{\partial f_t} \right) \quad (30)$$

$$0 = - \left(1 - \frac{1}{\Theta_{kt+1}} \left(\frac{\partial \mathcal{B}_{t+1}}{\partial b_t} + k_{t+1} \frac{\partial \Theta_{kt+1}}{\partial b_t} \right) \right) u_c(\mathcal{C}_t^1) + \phi u_c(\mathcal{C}_t^2) \quad (31)$$

where the total differentiation of backward transfers and rental price of capital with respect to current forward transfers are equal to $\frac{\partial \mathcal{B}_{t+1}}{\partial f_t} \equiv \frac{d\mathcal{B}_{t+1}}{dk_{t+1}} \mathcal{K}_{f_t} + \frac{d\mathcal{B}_{t+1}}{dh_{t+1}} \varphi_{\tilde{f}_t}$ and $\frac{\partial \Theta_{kt+1}}{\partial f_t} \equiv \Theta_{kk} \mathcal{K}_{f_t} + \Theta_{kh} \varphi_{\tilde{f}_t}$, respectively. Whereas the corresponding total differentiation with respect to backward transfers are equal to $\frac{\partial \mathcal{B}_{t+1}}{\partial b_t} \equiv \frac{d\mathcal{B}_{t+1}}{dk_{t+1}} \mathcal{K}_{b_t}$ and $\frac{\partial \Theta_{kt+1}}{\partial b_t} \equiv \Theta_{kk} \mathcal{K}_{b_t}$, respectively.

Let us first refer to Eq. (30). At each time an interior solution for the forward productive transfer is simply determined as an intertemporal maximization of the current adults' utility, who reap benefits and sustain costs by a variation in the current public expenditure. No intergenerational conflicts affect the setting of f_t . Since tax levying on labor income makes adults sustain the whole tax burden, the first term captures the adults' marginal cost caused by a positive variation on the fiscal dimension. The second term represents the expected marginal impact on the amount of next-period backward pork-barrel transfer due to a current variation in productive expenditure through the channels of both human and physical capital. Finally, the third term measures the expected marginal impact of f_t on the utility of next-period elderly through the increase in the rental price of capital. The redistributive choices are made as the outcome of a weighted bargaining between current adults and elderly. On one hand, an increase in backward transfers makes current adults to sustain direct costs and to either enjoy benefits or sustain costs from expected variations of both future pork-barrel transfers and the return to capital through the channel of physical capital, represented by the first part of Eq. (31). On the other hand, it makes old enjoy direct benefits from a current positive variation of b_t .

Even in absence of fiscal or political distortions, the efficient allocation is not achievable in equilibrium. Indeed, the existence of credit market constraints generates simultaneously two sources of distortions in equilibrium. On one hand, it precludes young from borrowing money for education investment, inducing underaccumulation of human capital. On the other hand, it reduces the investment portfolio for adults, determining overaccumulation of physical capital.¹¹ The provision of public education transfers through the institution of electoral competition partially offsets the inefficiency generated by the credit market conditions. However the gains

¹⁰By $\frac{dx}{dz}$ we denote the partial derivative of x with respect to z , and with $\frac{\partial x}{\partial z}$ the total differentiation.

¹¹See Boldrin and Montes (2005) for a complete analyses of distortions generated by credit market constraints in three period OLG economy with human capital.

in efficiency obtained in the politico-economic equilibrium compared to the autarchic allocation strongly depends on the voting institution which characterizes the political environment. In order to clearly show the positive relation between intergenerational political disagreement and economic efficiency, let us consider the following three cases.

Case I: No human capital and median voter framework, i.e. $f_t = 0$ and $\phi = 0$.¹² For simplicity suppose exogenous prices. From Eq. (30) and (31), it is easy to show that the first order condition to be satisfied requires $1 - \frac{1}{\Theta_{k_{t+1}}} \frac{dB_{t+1}}{dk_{t+1}} \mathcal{K}_{b_t} = 0$. Given that $\mathcal{K}_{b_t} < 0$, in equilibrium agents may sustain positive transfers to the elderly by coordinating on the level of aggregate physical capital. The lower the level of savings due to income taxes paid when adult, the higher the level of future benefits when old, i.e. $\frac{dB_{t+1}}{dk_{t+1}} < 0$. The intergenerational plan implemented by the median voter may also improve the economic overall efficiency starting from a condition of dynamic inefficiency.

Case II: Human capital and median voter framework, i.e. $f_t > 0$ and $\phi = 0$. When the productive human capital channel is introduced the politico-economic equilibrium outcome in the case of median voter slightly changes. Agents no longer coordinate as to the level of physical capital. The relevant state variable is now the total return to capital, which is affected by both human and physical capital. Adults have incentives to invest in educating the young in order to accumulate human capital and increase the next period return to capital. At the same time they do not have incentives to transfer public resource backward in terms of pension benefits. Indeed, adults perfectly anticipate that when old they are prevented to grab public resources by exerting political power. Furthermore, the next generation's promise concerning pension transfers are not credible because the return to capital will be positively affected by the previous period human capital investment. Consequently, in equilibrium public education investment will be financed by taxes paid by elderly and, if the physical capital productivity is high enough, such transfers will be also used to subsidize adults' consumption. Because of the resulting overaccumulation of both physical and human capital the economy will still be characterized by dynamic inefficiency.

Case III: Human capital and no median voter framework, i.e. $f_t \geq 0$ and $\phi > 0$. When the human capital investment is feasible, the existence of intergenerational conflicts over the redistribution of public resources improves the overall economic efficiency. Given that elderly actively participate to the political debate, they always gain positive transfers by exerting political power. At the same time they correct the overaccumulation of both physical and human capital.

In the following section we derive a closed form characterization of the politico-economic equilibrium, which solves the system of partial differential equations given by Eq. (30) and (31). Furthermore, we provide a quantitative measure of how political disagreement may affect economic efficiency in the long run.

¹²See Azariadis and Galasso (2002) for the case of exogenous prices and Forni (2005) for the case of endogenous prices.

5.1 Example Economy

Let us consider the following parametric case. Preferences are log-additive, i.e. $\log(c_t^1) + \beta \log(c_{t+1}^2)$, and the final good production and the human capital technology are respectively equal to:

$$y_t = Ak_t^\alpha h_t^{1-\alpha} \quad (32)$$

$$h_{t+1} = Bf_t^\eta h_t^{1-\eta} \quad (33)$$

with $A, B > 0$ and $\alpha, \eta \in (0, 1)$. Note that in balanced growth path the optimal education externality is simply equal to $\varepsilon = \delta(1 - \eta)$.¹³ Solving for the first best allocation, according to Definition 6, the growth rate of the economy and the rental price of capital per efficient unit of time turn out to be equal to, respectively:

$$\gamma^o \equiv \varphi^o(\tilde{f}^*; \delta) = \left((A^\eta B^{1-\alpha}) \mu_o^{(1-\alpha)\eta} \zeta_o^{\alpha\eta} \right)^{\frac{1}{1-\alpha(1-\eta)}} \quad (34)$$

$$R^o \equiv \Theta_k^o(\tilde{k}^*; \delta) = \alpha \left((A^\eta B^{1-\alpha}) \mu_o^{(1-\alpha)\eta} \zeta_o^{\alpha-1} \right)^{\frac{1}{1-\alpha(1-\eta)}} \quad (35)$$

where $\mu_o \equiv \frac{\eta\delta(1-\alpha)}{1-\varepsilon}$ and $\zeta_o \equiv \frac{(1-\varepsilon)-\eta\delta(1-\alpha)-(1-\delta)(1-\varepsilon\alpha)}{(1-\varepsilon)}$. Both \tilde{k}^* and \tilde{f}^* are increasing functions in the dynasty discount factor δ and converge to zero when δ goes to zero.

We now evaluate how the politico-economic equilibrium, as described in Definition 8, performs with respect to the Pareto optimal allocation resumed by Eq. (34) and (35).

Lemma 1 *If a Markov subgame perfect politico-economic equilibrium exists, then the following condition must hold:*

$$\varphi_{\tilde{f}_t} - \frac{\vartheta_{\tilde{k}}(\tilde{k}_{t+1})}{\vartheta(\tilde{k}_{t+1}) - \tilde{k}_{t+1}\vartheta_{\tilde{k}}(\tilde{k}_{t+1})} \Lambda(\phi) = 0 \quad (36)$$

where $\Lambda(\phi) = \frac{(1+\beta\alpha)(\phi+(1+\beta(\alpha+\eta(1-\alpha))))}{\phi+\alpha\beta(\phi+(1+\beta(\alpha+\eta(1-\alpha))))} > 1$ and $\Lambda_\phi(\cdot) < 0$.

Proof. (See appendix). ■

Lemma 1 characterizes the necessary condition for the existence of the politico-economic equilibrium. Furthermore, Eq. (36) is directly comparable to the education optimal wedge, Eq. (10). Note that $\Lambda(0) = \max \Lambda(\phi)$, which implies that the lower ϕ , the larger the difference with respect to the first best allocation and, in turns, the inefficiency.

Under Lemma 1, by applying Definition 2 the following proposition holds.

Proposition 2 *A unique Markov subgame perfect politico-economic equilibrium which sustains the policies $\mathcal{F}(h_t, k_t) = \xi^f y_t$ and $\mathcal{B}(h_t, k_t) = \xi^b y_t$ exists, where $\xi^f \equiv \frac{\beta\eta(1-\alpha)}{\phi+(1+\beta(\alpha+\eta(1-\alpha)))}$ and $\xi^b \equiv \frac{\phi(1-\alpha)-\alpha(1+\beta(\alpha+\eta(1-\alpha)))}{\phi+(1+\beta(\alpha+\eta(1-\alpha)))}$.*

Proof. (See appendix). ■

¹³For detailed calculus see Appendix.

By solving via backward, taking a finite time horizon economy and making the time goes to infinite, the incentive structure of the game emerges clearly. As long as $\phi > \frac{\alpha}{1-\alpha}$, in the last period of the economy the elderly are able to grab a share of the total production through both capitalized savings and pension contributions. As a consequence, when adults, agents have incentives to invest a fraction of the collected taxes in public education.

Solving for the politico-economic equilibrium, the growth rate of the economy and the rental price of capital per efficient unit of time are equal to:

$$\gamma^p \equiv \varphi^p(\tilde{f}; \phi) = \left((A^\eta B^{1-\alpha}) \mu_p^{(1-\alpha)\eta} \zeta_p^{\alpha\eta} \right)^{\frac{1}{1-\alpha(1-\eta)}} \quad (37)$$

$$R^p \equiv \Theta_k^p(\tilde{k}; \phi) = \alpha \left((A^\eta B^{1-\alpha}) \mu_p^{(1-\alpha)\eta} \zeta_p^{\alpha-1} \right)^{\frac{1}{1-\alpha(1-\eta)}} \quad (38)$$

where $\mu_p \equiv \frac{\beta\eta(1-\alpha)}{\phi+(1+\beta(\alpha+\eta(1-\alpha)))}$ and $\zeta_p \equiv \frac{(1+\alpha\beta)\alpha\beta}{\alpha\beta(1+\beta(\alpha+\eta(1-\alpha))+\phi)+\phi}$. In order to fully understand the role of intergenerational political disagreement measured by ϕ , let us consider the case $\phi = 0$, which is equivalent to consider a framework where the median-voter belongs to the adults' cohort.

Corollary 1 *When the median voter is the adult, the rental price of capital and the economy's growth rate are respectively equal to $\Theta_k^m(\tilde{k}) = \lim_{\phi \rightarrow 0} \Theta_k^p(\tilde{k}; \phi)$ and $\varphi^m(\tilde{f}) = \lim_{\phi \rightarrow 0} \varphi^p(\tilde{f}; \phi)$, such that $\varphi^m(\tilde{f}) > \varphi^p(\tilde{f}; \phi)$ and $\Theta_k^m(\tilde{k}) < \Theta_k^p(\tilde{k}; \phi)$.*

Proof. (See appendix). ■

As argued in section 5, in our economy the absence of intergenerational conflicts is detrimental for the economy's efficiency. If the median voter is the adult in equilibrium he will vote for a multidimensional fiscal platform characterized by: i) negative transfers from the elderly, ii) possible transfers to adults in order to subsidize consumption, and iii) high investment in public education. Specifically, if the share of capital over total production is large enough, i.e. $-\alpha^2(\beta(1-\eta)) - \alpha(1+2\eta\beta) + \eta\beta < 0$, taxes paid by the elderly are used both to finance public education and to subsidize adults' consumption. As a consequence the economy will be characterized by overaccumulation of both physical and human capital.

As soon as a share of political power is assigned to the elderly, the equilibrium political platform reverses, turning out to be characterized by: i) positive transfers to the elderly, ii) negative transfers from the adult, iii) lower investment in public education. The participation of the elderly in the political debate and the consequent emergence of intergenerational political disagreement on the redistribution of public resources improves the overall economic efficiency.

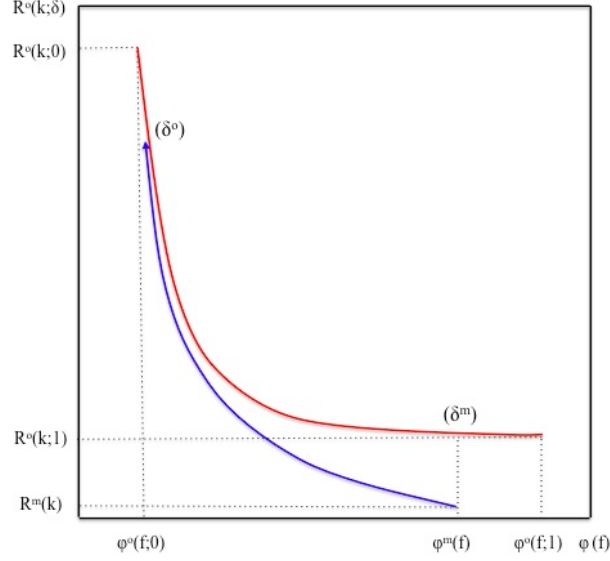


Fig. 1: The parametric values are $A=0.2$, $B=0.2$, $\alpha=0.4$, $\eta=0.2$ and $\beta=0.95^{30}$

Lemma 2 For any level of welfare weights, $\delta \in (0, \delta^m)$ with $\delta^m > 0$, $\Theta_k^m(\tilde{k}) < \Theta_k^o(\tilde{k}^*; \delta)$ and $\varphi^m(\tilde{f}) > \varphi^o(\tilde{f}^*; \delta)$.

Proof. (See appendix). ■

Lemma 2 states that the equilibrium allocation achieved in the median voter case is always suboptimal and induces overaccumulation of both human capital and physical capital for any welfare weight $\delta \in (0, \delta^m)$. For illustrative purpose, let us consider a specific quantitative exercise. In Fig. 1 are reported both the pair of optimal rental price of capital and economy's growth rate as a function of δ (red curve), and the pair of rental price of capital and economy's growth rate obtained in the politico-economic equilibrium as a function of ϕ (blue curve). As shown in Fig. 1, when $\phi = 0$, the economy experiences a higher level of both human capital and physical capital accumulation with respect to the first best allocation for any $\delta \leq \delta^m$.

Using Lemma 2 the following Proposition holds.

Proposition 3 For any $\epsilon > 0$, a level $\delta^o \in (0, 1)$ and $\hat{\phi} > 0$ such that:

$$\lim_{\phi \rightarrow \hat{\phi}} \left| \Theta_k^p(\tilde{k}; \phi) - \Theta_k^o(\tilde{k}^*; \delta^o) \right| < \epsilon$$

$$\lim_{\phi \rightarrow \hat{\phi}} \left| \varphi^p(\tilde{k}; \phi) - \varphi^o(\tilde{k}^*; \delta^o) \right| < \epsilon$$

exist.

Proof. (See appendix). ■

From Proposition 3 it follows that the higher the elderly political power and, consequently, the higher the political disagreement, the lower the accumulation of human capital, the lower

the accumulation of physical capital and the higher the efficiency for a given welfare weight. This gain in efficiency is reported in the Fig. 1 and is represented by reduction of the distance between the blue curve and the red one for a given level of welfare weight.

6 Distortionary Case

In this section we consider two further sources of distortions, which might affect the politico-economic equilibrium determined in the previous section. First, we introduce capital taxation (fiscal distortions). Second, we enable parties to take care of their ideological positions (political distortions). We analyze the two cases separately.

6.1 Capital Taxation

Let us introduce distortionary capital taxation. Eq. (5) holds and public expenditure is financed through both labor and capital income taxes.

Proposition 4 *A unique Markov perfect politico-economic equilibrium characterized by parties \mathcal{L}_t and \mathcal{R}_t proposing the same political platform, $z_{\mathcal{L}_t} = z_{\mathcal{R}_t} = z_t$ exists. Let $f_{t+1} = \mathcal{F}(k_{t+1}, h_{t+1})$ and $b_{t+1} = \mathcal{B}(k_{t+1}, h_{t+1})$ be the equilibrium policy rules, the parties maximization program is:*

$$\begin{aligned} \max_{\{b_t, f_t, h_{t+1}, k_{t+1}\}} & \phi u(C^2(b_t, f_t, h_t, k_t)) + u(C^1(f_t, b_t, h_t, k_t, k_{t+1})) \\ & + \beta u(C^2(\mathcal{B}_{t+1}, \mathcal{F}_{t+1}, h_{t+1}, k_{t+1})) \end{aligned}$$

under the constraint:

$$u_c(C_t^1) - \beta(\Theta_{t+1} - \mathcal{B}_{t+1} - \mathcal{F}_{t+1}) \frac{\Theta_{kt+1}}{\Theta_{t+1}} u_c(C_{t+1}^2) = 0 \quad (39)$$

Proof. (See appendix). ■

Two main elements distinguish this statement from Proposition 1. First, the elderly consumption turns out to be negatively affected by the amount of provided education transfers. Larger the implemented forward transfers, higher the income tax rate charged to both adults and elderly. Second, the economic Euler condition, which constrains the parties' maximization program, is affected by the expected capital taxation. It implies that in equilibrium the saving wedge, Eq. (9), is different from zero, capturing the fiscal distortion.

By applying envelope theorem and using Eq. (16), (17), and (39), the Euler conditions with respect to the policies rules f_t and b_t are equal to, respectively:

$$\begin{aligned} 0 = & -\frac{\Theta_{ht} h_t}{\Theta_t} u_c(C_t^1) - \underbrace{\phi \frac{\Theta_{kt} k_t}{\Theta_t} u_c(C_t^2)}_{(1^*)} + \frac{1}{\Theta_{kt+1}} \left(\frac{1}{1-\pi_{t+1}} \frac{\partial \mathcal{B}_{t+1}}{\partial f_t} + k_{t+1} \frac{\partial \Theta_{kt+1}}{\partial f_t} \right) u_c(C_t^1) \\ & - \underbrace{\frac{k_{t+1}}{1-\pi_{t+1}} \left(\frac{\partial \tilde{\mathcal{B}}_{t+1}}{\partial f_t} + \frac{\partial \tilde{\mathcal{F}}_{t+1}}{\partial f_t} \right)}_{(2^*)} u_c(C_t^1) \end{aligned} \quad (40)$$

$$\begin{aligned}
0 = & -\frac{\Theta_{h_t h_t}}{\Theta_t} (u_c(C_t^1) - \phi u_c(C_t^2)) + \frac{1}{\Theta_{k_{t+1}}} \left(\frac{1}{1-\pi_{t+1}} \frac{\partial \mathcal{B}_{t+1}}{\partial b_t} + k_{t+1} \frac{\partial \Theta_{k_{t+1}}}{\partial b_t} \right) u_c(C_t^1) \\
& - \underbrace{\frac{k_{t+1}}{1-\pi_{t+1}} \left(\frac{\partial \tilde{\mathcal{B}}_{t+1}}{\partial b_t} + \frac{\partial \tilde{\mathcal{F}}_{t+1}}{\partial b_t} \right)}_{(3^*)} u_c(C_t^1)
\end{aligned} \tag{41}$$

where $\tilde{\mathcal{B}}_{t+1}(\cdot)$ and $\tilde{\mathcal{F}}_{t+1}(\cdot)$ denote the backward and forward transfer functions per GDP. Furthermore, the total differentiation with respect to current current policies of future expected transfers are equal to $\frac{\partial \tilde{\mathcal{B}}_{t+1}}{\partial f_t} \equiv \frac{d\tilde{\mathcal{B}}_{t+1}}{dk_{t+1}} \mathcal{K}_{f_t} + \frac{d\tilde{\mathcal{B}}_{t+1}}{dh_{t+1}} \varphi_{\tilde{f}_t}$, $\frac{\partial \tilde{\mathcal{F}}_{t+1}}{\partial f_t} \equiv \frac{d\tilde{\mathcal{F}}_{t+1}}{dk_{t+1}} \mathcal{K}_{f_t} + \frac{d\tilde{\mathcal{F}}_{t+1}}{dh_{t+1}} \varphi_{\tilde{f}_t}$, $\frac{\partial \tilde{\mathcal{B}}_{t+1}}{\partial b_t} \equiv \frac{d\tilde{\mathcal{B}}_{t+1}}{dk_{t+1}} \mathcal{K}_{b_t}$ and $\frac{\partial \tilde{\mathcal{F}}_{t+1}}{\partial b_t} \equiv \frac{d\tilde{\mathcal{F}}_{t+1}}{dk_{t+1}} \mathcal{K}_{b_t}$. Compared to the Eq. (30) and (31), distortionary taxation on capital affects the politico-economic equilibrium in three ways. Let us first refer to Eq. (40). Differently from the no distortionary case, old sustain the fiscal burden together with the adults, (1*). Consequently, intergenerational conflicts affect the setting of forward transfers. The term (2*) captures the quantitative impact of the fiscal distortion on the next-period utility of the current living adults. In equilibrium the influence might be either positive or negative, depending on the relation between current education transfers and future income tax rate. When negative, the incentives to invest in human capital are dampened, even if the elderly are endowed with relatively small political power. The adults anticipate that an increase in human capital investment will prompt a fiscal adjustment that, in turns, will increase their future fiscal burden. As a consequence, they decide to invest less in education, partially correcting the overaccumulation of human capital. The term (3*) in Eq. (41) describes the role of the fiscal distortion on the next-period utility through the backward channel. Equivalently to the previous case, it might be either positive or negative. When positive, the parties have higher incentives to redistribute backward, dampening the overaccumulation of physical capital. As an insight, like the presence of political institutions that foster the intergenerational conflict, the distortionary capital taxation may improve dynamic efficiency.

6.2 Ideological Competition

In this section we analyze ideological competition among parties. Forward-looking and ideologically heterogeneous politicians are able to strategically manipulate their probability of winning future election through the current implemented fiscal platform.

Proposition 5 *The Markov perfect politico-economic equilibrium is characterized by parties \mathcal{L}_t and \mathcal{R}_t proposing different political platform, $z_{\mathcal{L}_t} \neq z_{\mathcal{R}_t}$. Let $f_{i_{t+1}} = \mathcal{F}_{i_{t+1}}(k_{t+1}, h_{t+1})$ and $b_{i_{t+1}} = \mathcal{B}_{i_{t+1}}(k_{t+1}, h_{t+1})$ be the equilibrium policy rules, the parties maximization program is:*

$$\begin{aligned}
\max_{\{b_{i_t}, f_{i_t}, h_{t+1}, k_{t+1}\}} & -(\psi^1 + \psi^2) l(z_{i_t} - \bar{z}_i) \\
& + \phi u(C_{i_t}^2(b_{i_t}, h_t, k_t)) + u(C_{i_t}^1(f_{i_t}, b_{i_t}, h_t, k_t, k_{t+1})) \\
& + \beta E[\mathcal{P}_{i_t}(W_{i_t}^2 + \theta_{t+1} + \psi_{j_{t+1}}^2) + (1 - \mathcal{P}_{i_t}) V_{i_t}^2]
\end{aligned}$$

under the constraint:

$$u_c(C_{i_t}^1) - \beta E^{\mathcal{P}_{i_t}} \left[\Theta_{k_{t+1}} u_c(C_{i_{t+1}}^2) \right] = 0 \tag{42}$$

Proof. (See appendix). ■

Departing from the cases discussed in the previous sections, the parties compete by jointly maximizing their probability to win election and minimizing the ideological distance of the implemented political platform from the ideal one, Eq. (26) and (27). Furthermore, agents internalize the uncertainty about the future elections outcome, which is conditioned by the political choices of the previous period incumbent party. The following first order condition for each party, i_t , characterizes the Nash equilibrium of the game:

$$0 = \underbrace{-\frac{(\psi^1 + \psi^2) \partial l(z_{i_t} - \bar{z}_i)}{\partial z_{i_t}}}_{\text{ideological competition}} + \frac{\psi^2 \partial u(c_{i_t}^2)}{\psi^1 \partial z_{i_t}} + \frac{\partial u(c_{i_t}^1)}{\partial z_{i_t}} + \beta E \underbrace{\left[\frac{\partial \mathcal{P}_{i_t}}{\partial z_{i_t}} (W_{i_t}^2 + \theta_{t+1} + \psi_{j_{t+1}}^2 - V_{i_t}^2) + \mathcal{P}_{i_t} \frac{\partial W_{i_t}^2}{\partial z_{i_t}} + (1 - \mathcal{P}_{i_t}) \frac{\partial V_{i_t}^2}{\partial z_{i_t}} \right]}_{\text{endogenous political turnover}}$$

The parties' ideological heterogeneity induces representatives to propose different political platforms. As a consequence, the equilibrium policy does not reduce to a common fiscal proposal, which maximizes a weighted sum of agents' utility. Furthermore, $\bar{\theta}(k_t, h_t)$ does not degenerate to zero and in equilibrium is a function of the next period threshold level, $\bar{\theta}(k_{t+1}, h_{t+1})$. As a result, the probability of being elected turns out to be neither constant nor equal to one half. Contrarily, it is a function of the relevant payoff state variables of the economy, $\mathcal{P}_{i_t}(k_{t+1}, h_{t+1})$. Perfect forward looking parties could act strategically by choosing a platform which maximize the probability of winning current elections, also taking into account the impact of current policies on the next period probability. The possibility to endogenously control for the political turnover by being in power adds the political distortion quantified by the third term of first order condition reported above.

7 Conclusions

In this paper we show how intergenerational conflicts over the redistribution of public resources may enhance efficiency when human capital accumulation is the main force driving the economic growth. We build a tractable dynamic politico-economic model in a neoclassical environment and determine the subgame Markov perfect equilibrium of a dynamic game characterized by both ideological heterogeneous voters and office-seeking political parties. Three main sources of distortions characterize the theoretical environment: Incomplete credit markets, physical capital taxation, and ideological political competition. When the credit market constraint is the only source of distortion, in equilibrium the political platform turns out to be characterized by: i) positive transfers to the elderly, ii) negative transfers from the adult, and iii) lower investment in public education. The elderly claim for positive transfers induces a simultaneous reduction in both physical and human capital accumulation, partially offsetting the dynamic inefficiency. Adopting a first order characterization, we test the robustness of our theoretical

findings. Specifically, we study how fiscal and political distortions, related to capital taxation and ideological political competition, respectively, alter the main finding about the positive correlation between intergenerational political disagreement and efficiency. We find that both types of distortions might be welfare improving from a dynamic efficiency point of view, dampening the positive impact of intergenerational conflicts on the overall economic welfare. In order to quantitatively assess the gains in efficiency and the loss due to distortions, we need to perform numerical analyses. Future works will be directed toward this line of research.

8 Appendix A

Derivation of closed form solution for efficient allocation

Due to the log-preferences, which cancel out income and substitution effects, savings are a fraction of current variables. Furthermore, along the balanced growth path we attain $\frac{u_{c_t^2}(c_t^{2*})}{u_{c_{t+1}^2}(c_{t+1}^{2*})} = \frac{c_{t+1}^{2*}}{c_t^{2*}} = \varphi(\tilde{f}^*)$.

Using the Euler conditions (8) and (9) we obtain:

$$\delta \vartheta_{\tilde{k}}(\tilde{k}^*) = \varphi(\tilde{f}^*) \quad (43)$$

The condition (10) in balanced growth path is $\varphi_{\tilde{f}}(\tilde{f}^*) = \frac{\vartheta_{\tilde{k}}(\tilde{k}^*)}{\vartheta(\tilde{k}^*) - \tilde{k}^* \vartheta_{\tilde{k}}(\tilde{k}^*)} (1 - \varepsilon)$ where $\varepsilon = \frac{1}{\vartheta_{\tilde{k}}(\tilde{k}^*)} (\varphi(\tilde{f}^*) - \tilde{f}^* \varphi_{\tilde{f}}(\tilde{f}^*))$. Using the condition (43) and Eq. (33) we have ε constant and equal to $\varepsilon = \delta(1 - \eta)$, consequently $\varphi_{\tilde{f}}(\tilde{f}^*)$ turns out to be equal to:

$$\varphi_{\tilde{f}}(\tilde{f}^*) = \frac{\vartheta_{\tilde{k}}(\tilde{k}^*)}{\vartheta(\tilde{k}^*) - \tilde{k}^* \vartheta_{\tilde{k}}(\tilde{k}^*)} (1 - \varepsilon) \quad (44)$$

Using the parametric forms given by Eq. (32) and (33) and plugging the partial derivatives into (44) we get $\tilde{f}^* = \frac{1-\alpha}{\alpha} \frac{\eta}{1-\varepsilon} \tilde{k}^* \varphi(\tilde{f}^*)$. Using Eq. (43) we get $\tilde{f}^* = \frac{1-\alpha}{\alpha} \frac{\eta \delta}{1-\varepsilon} \tilde{k}^* \vartheta_{\tilde{k}}(\tilde{k}^*)$ and finally:

$$\tilde{f}^* = \frac{\eta \delta (1 - \alpha)}{1 - \varepsilon} \tilde{y}^* \quad (45)$$

Aggregate consumption along the balanced growth path is then equal to:

$$\tilde{C}^* \equiv \tilde{c}^{1*} + \tilde{c}^{2*} = \tilde{y}^* - \underbrace{\frac{h_{t+1}^*}{h_t^*} \tilde{k}^* - \tilde{f}^*}_{\tilde{s}^*} \quad (46)$$

where h^* is the human capital level in steady state. After using Eq. (43) and (45) and rearranging, we obtain that aggregate consumption is also equal to $\tilde{C}^* = \frac{(1-\delta)(1-\varepsilon\alpha)}{(1-\varepsilon)} \tilde{y}^*$. To identify how aggregate consumption is shared between the first and the second period let us recall (9) in balanced path:

$$\vartheta_{\tilde{k}}(\tilde{k}^*) = \frac{u_{c_t^1}(c_t^{1*})}{\beta u_{c_{t+1}^2}(c_{t+1}^{2*})} = \frac{\varphi(\tilde{f}^*)}{\beta} \frac{\tilde{c}^{2*}}{\tilde{c}^{1*}}$$

then we yield $\frac{\tilde{c}^{2*}}{\tilde{c}^{1*}} = \frac{\beta}{\delta}$ and consequently $\tilde{c}^{1*} = \frac{\delta}{\delta+\beta} \tilde{C}^*$ and $\tilde{c}^{2*} = \frac{\beta}{\delta+\beta} \tilde{C}^*$ and individual savings are:

Let us denote with $\mu_o \equiv \frac{\eta \delta (1 - \alpha)}{1 - \varepsilon}$, $\zeta_o \equiv \frac{(1-\varepsilon) - \eta \delta (1 - \alpha) - (1-\delta)(1-\varepsilon\alpha)}{(1-\varepsilon)}$. Then using Eq. (46) we

finally obtain the steady state level of \tilde{k}^* :

$$\begin{aligned}\tilde{k}^* &= \left(\frac{A}{B} \frac{\zeta_o}{(A\mu_o)^\eta} \right)^{\frac{1}{1-\alpha(1-\eta)}}, & \tilde{y}^* &= A \left(\frac{A}{B} \frac{\zeta_o}{(A\mu_o)^\eta} \right)^{\frac{\alpha}{1-\alpha(1-\eta)}} \\ \tilde{f}^* &= A\mu_o \left(\frac{A}{B} \frac{\zeta_o}{(A\mu_o)^\eta} \right)^{\frac{\alpha}{1-\alpha(1-\eta)}}, & \Theta_k^o(\tilde{k}^*) &= \alpha A \left(\frac{A}{B} \frac{\zeta_o}{(A\mu_o)^\eta} \right)^{\frac{\alpha-1}{1-\alpha(1-\eta)}}\end{aligned}$$

Finally, using Eq. (33) and (45), the economy's growth rate turns out to be equal to:

$$\gamma^o \equiv \varphi(\tilde{f}^*) = B (A\mu_o)^\eta \left(\frac{A}{B} \frac{\zeta_o}{(A\mu_o)^\eta} \right)^{\frac{\alpha\eta}{1-\alpha(1-\eta)}}$$

9 Appendix B

Proposition (1). Let us consider a finite horizon economy, which ends at time $T = 2$. Parties are not ideologically heterogenous. In the last period the political maximization program for each party i_t is equal to:

$$\max_{z_{i_2}} \frac{\psi^2}{\psi^1} u(C_{i_2}^2) + u(C_{i_2}^1)$$

Given that the parties' objective function is simply the probability of winning elections, then in equilibrium they propose the same platform, $z_{\mathfrak{R}_2} = z_{\mathcal{L}_2}$ and $\Delta u(C_2^2) = \Delta u(C_2^1) = 0$. Using Eq. (23), it follows that $\bar{\theta}_2 = 0$ and $\Pr(\theta_2 > \bar{\theta}_2) = \Pr(\theta_2 < \bar{\theta}_2) = \frac{1}{2}$.

At time $T = 1$ the maximization program for party i_1 is:

$$\max_{z_{i_1}} \frac{\psi^2}{\psi^1} u(C_{i_1}^2) + u(C_{i_1}^1) + \beta E(\mathcal{P}_{i_1}(W_{i_1}^2 + \theta_2 + \psi_{j_2}^2) + (1 - \mathcal{P}_{i_1})V_{i_2}^2) \quad (47)$$

where $\mathcal{P}_{\mathfrak{R}_1} = \frac{1}{2} - \theta\bar{\theta}_2$ and $\mathcal{P}_{\mathcal{L}_1} = \frac{1}{2} + \theta\bar{\theta}_2$. Given that $\bar{\theta}_2$ is equal to 0 independently from the incumbent party at time $t = 1$, then the conditional probability of being elected turns out to be equal to:

$$\mathcal{P}_{\mathfrak{R}_1} = \mathcal{P}_{\mathcal{L}_1} = \frac{1}{2}$$

The maximization program given by Eq. (47) reduces to:

$$\max_{z_{i_1}} \frac{\psi^2}{\psi^1} u(C_{i_1}^2) + u(C_{i_1}^1) + \beta u(C_{i_2}^2)$$

Replicating the argument at time $T = 2$, we have $z_{\mathfrak{R}_1} = z_{\mathcal{L}_1}$ with $\Delta u(C_1^2) = \Delta u(C_1^1) = 0$ and $\mathcal{V}^2(k_2, h_2; \mathcal{L}_1) - \mathcal{W}^2(k_2, h_2; \mathfrak{R}_1) = 0$. Using Eq. (23), $\bar{\theta}_1 = 0$ and $\Pr(\theta_1 > \bar{\theta}_1) = \Pr(\theta_1 < \bar{\theta}_1) = \frac{1}{2}$. Hence the two candidates' platform converges in equilibrium to the same fiscal policy that maximizes a weighted utility of current adults and old:

$$\max_{z_t} \frac{\psi^2}{\psi^1} u(C^2(b_t, h_t, k_t)) + u(C^1(f_t, b_t, h_t, k_t, k_{t+1})) + \beta u(C^2(b_{t+1}, h_{t+1}, k_{t+1}))$$

under the constraint $u_c(C_t^1) - \beta\Theta_{kt+1}u_c(C_{t+1}^2) = 0$. ■

Lemma (1). Let us consider a simple two period-economy, $T = 2$, and solve backward. At the last period $k_2 = 0$ and the politicians maximization problem is:

$$\max_{f_2, b_2} u(C_2^1) + \phi u(C_2^2)$$

In the last period $f_2 = 0$ and the Euler condition which determines the amount of pension is $\frac{u_{C_2^1}}{u_{C_2^2}} = \phi$ which implies $b_2 = \frac{\phi(1-\alpha)-\alpha}{1+\phi}y_2$. At time $t = 1$ the maximization program becomes:

$$\max_{f_1, b_1} u(C_1^1) + \beta u(C_2^2) + \phi u(C_1^2)$$

After some algebra the politicians' Euler conditions turns out to be equal to:

$$\frac{u_{\mathcal{C}_1^2}}{u_{\mathcal{C}_1^1}} = \frac{1 + \alpha\beta}{\phi + \alpha\beta(1 + \phi)}$$

$$\frac{u_{\mathcal{C}_1^2}}{u_{\mathcal{C}_1^1}} = \frac{1}{1 + \phi} \varphi_{\tilde{f}_1} \frac{\vartheta(\tilde{k}_2) - \tilde{k}_2 \vartheta_{\tilde{k}}(\tilde{k}_2)}{\vartheta_{\tilde{k}}(\tilde{k}_2)}$$

Recalling that $\varphi_{\tilde{f}_1} \frac{\vartheta(\tilde{k}_2) - \tilde{k}_2 \vartheta_{\tilde{k}}(\tilde{k}_2)}{\vartheta_{\tilde{k}}(\tilde{k}_2)} = \frac{k_2}{f_1} \frac{\eta(1-\alpha)}{\alpha}$, solving the system we have at time $t = 1$:

$$f_1 = \frac{\beta\eta(1-\alpha)}{\phi + (1 + \beta(\alpha + \eta(1-\alpha)))} y_1$$

$$b_1 = \frac{\phi(1-\alpha) - \alpha(1 + \beta(\alpha + \eta(1-\alpha)))}{\phi + (1 + \beta(\alpha + \eta(1-\alpha)))} y_1$$

Due to the specific parametric form, after only two recursions the policies remain unchanged and the political Euler conditions becomes equal to:

$$\frac{u_{\mathcal{C}_t^2}}{u_{\mathcal{C}_t^1}} = \frac{1 + \beta\alpha}{\phi + \alpha\beta(\phi + (1 + \beta(\alpha + \eta(1-\alpha))))}$$

$$\frac{u_{\mathcal{C}_t^2}}{u_{\mathcal{C}_t^1}} = \frac{1}{\phi + (1 + \beta(\alpha + \eta(1-\alpha)))} \frac{F_{h_t}}{F_{k_t}} \varphi_{\tilde{f}_t}$$

which implies:

$$\varphi_{\tilde{f}_t} - \frac{\vartheta_{\tilde{k}}(\tilde{k}_{t+1})}{\vartheta(\tilde{k}_{t+1}) - \tilde{k}_{t+1} \vartheta_{\tilde{k}}(\tilde{k}_{t+1})} \Lambda(\phi) = 0$$

where $\Lambda(\phi) = \frac{(1+\beta\alpha)(\phi+(1+\beta(\alpha+\eta(1-\alpha))))}{\phi+\alpha\beta(\phi+(1+\beta(\alpha+\eta(1-\alpha))))} > 1$ and $\Lambda'(\phi) = -\frac{(1+\alpha\beta)(1+\alpha\beta(1-\eta)+\beta\eta)}{(\phi+\alpha\beta(1+\beta(\alpha+\eta(1-\alpha))+\phi^2))} < 0$. ■

Proposition (2). Using Lemma 1, rearranging and solving for f_t and b_t we obtain:

$$\mathcal{F}(h_t, k_t) = \xi^f A k_t^\alpha h_t^{1-\alpha}, \quad \mathcal{B}(h_t, k_t) = \xi^b A k_t^\alpha h_t^{1-\alpha}$$

with $\xi^f \equiv \frac{\beta\eta(1-\alpha)}{\phi+(1+\beta(\alpha+\eta(1-\alpha)))}$ and $\xi^b \equiv \frac{\phi(1-\alpha)-\alpha(1+\beta(\alpha+\eta(1-\alpha)))}{\phi+(1+\beta(\alpha+\eta(1-\alpha)))}$ and, by using condition (5), the income tax rate is equal to $\pi_t = \frac{\xi^f + \xi^b}{1-\alpha} = 1 - \frac{1+\alpha\beta}{(1-\alpha)(1+\beta(\alpha+\eta(1-\alpha))+\phi)}$. If $\phi > \frac{(1+\alpha\beta)\alpha}{1-\alpha} - (1-\alpha)\beta\eta$ then $\pi_t > 0$.

Furthermore, the laws of motion of the state variables are:

$$k_{t+1} = \frac{A(1+\alpha\beta)\alpha\beta}{\alpha\beta(1+\beta(\alpha+\eta(1-\alpha))+\phi)+\phi} k_t^\alpha h_t^{1-\alpha} \quad (48)$$

$$h_{t+1} = B \left(\frac{A\beta\eta(1-\alpha)}{\phi+(1+\beta(\alpha+\eta(1-\alpha)))} \right)^\eta k_t^{\alpha\eta} h_t^{1-\alpha\eta} \quad (49)$$

In balanced growth path \tilde{k}_{t+1} is constant over time. Let us denote with $\mu_p \equiv \frac{\beta\eta(1-\alpha)}{\phi+(1+\beta(\alpha+\eta(1-\alpha)))}$, $\zeta_p \equiv \frac{(1+\alpha\beta)\alpha\beta}{\alpha\beta(1+\beta(\alpha+\eta(1-\alpha))+\phi)+\phi}$. Then, by using Eq. (48) and (49) and rearranging, we obtain the

steady state level of the politico-economic allocation is equal to:

$$\tilde{f} = A\mu_p \left(\frac{A}{B} \frac{\zeta_p}{(A\mu_p)^\eta} \right)^{\frac{\alpha}{1-\alpha(1-\eta)}}, \quad \tilde{k} = \left(\frac{A}{B} \frac{\zeta_p}{(A\mu_p)^\eta} \right)^{\frac{1}{1-\alpha(1-\eta)}}, \quad \tilde{y} = A \left(\frac{A}{B} \frac{\zeta_p}{(A\mu_p)^\eta} \right)^{\frac{\alpha}{1-\alpha(1-\eta)}}$$

Then, the economy's growth rate and the rental price of capital are, respectively:

$$\begin{aligned} \gamma^p &\equiv \varphi^p(\tilde{k}; \phi) = \left((A^\eta B^{1-\alpha}) \mu_p^{(1-\alpha)\eta} \zeta_p^{\alpha\eta} \right)^{\frac{1}{1-\alpha(1-\eta)}} \\ R^p &\equiv \Theta_k^p(\tilde{k}; \phi) = \alpha \left((A^\eta B^{1-\alpha}) \mu_p^{(1-\alpha)\eta} \zeta_p^{\alpha-1} \right)^{\frac{1}{1-\alpha(1-\eta)}} \end{aligned}$$

■

Corollario (1). From Lemma 1, taking $\phi = 0$ and solving backward, we obtain $\mathcal{F}_m(h_t, k_t) = \xi_m^f y_t$ and $\mathcal{B}_m(h_t, k_t) = \xi_m^b y_t$ with $\xi_m^f \equiv \frac{\beta\eta(1-\alpha)}{1+\beta(\alpha+\eta(1-\alpha))}$ and $\xi_m^b \equiv -\frac{\alpha(1+\beta(\alpha+\eta(1-\alpha)))}{1+\beta(\alpha+\eta(1-\alpha))}$. Then, by using condition (5), the income tax rate is equal to $\pi_t = 1 - \frac{1+\alpha\beta}{(1-\alpha)(1+\beta(\alpha+\eta(1-\alpha)))}$. Furthermore, the rental price of capital, $R^m(\tilde{k})$, and the economy's growth rate, $\varphi^m(\tilde{k})$, in the median voter case are equal to:

$$\begin{aligned} \Theta_k^m(\tilde{k}) &= \lim_{\phi \rightarrow 0} \Theta_k^p(\tilde{k}; \phi) \\ \varphi^m(\tilde{k}) &= \lim_{\phi \rightarrow 0} \varphi^p(\tilde{k}; \phi) \end{aligned}$$

such that $\varphi^m(\tilde{k}) > \varphi^p(\tilde{k}; \phi)$ and $\Theta_k^m(\tilde{k}) < \Theta_k^p(\tilde{k}; \phi)$. ■

Lemma (2). Using Eq. (34), (35), (37) and (38), we have that $\Theta_k^m(\tilde{k}) < \Theta_k^o(\tilde{k}^*; \delta)$ when $\frac{\mu_m}{\mu_o} < \left(\frac{\zeta_m}{\zeta_o} \right)^{\frac{1}{\eta}}$, and $\varphi^m(\tilde{f}) > \varphi^o(\tilde{f}^*; \delta)$ when $\frac{\mu_m}{\mu_o} > \left(\frac{\zeta_o}{\zeta_m} \right)^{\frac{1-\alpha}{1-\alpha}}$. The joint conditions imply $\zeta_o < \zeta_m$. Plugging the values for ζ_o and ζ_m we obtain that the Lemma holds for any $\delta < \bar{\delta} = \min \left\{ \frac{1+\alpha\beta}{\alpha(1+\beta(\alpha+\eta(1-\alpha)))}, 1 \right\}$. ■

Proposition (3). From Lemma 2, for any level of dynastic welfare weight $\delta < \bar{\delta}$, we have:

$$\begin{aligned} \Theta_k^m(\tilde{k}) - \Theta_k^o(\tilde{k}^*; \delta) &< 0 \\ \varphi^m(\tilde{k}) - \varphi^o(\tilde{k}^*; \delta) &> 0 \end{aligned}$$

Furthermore, according to Corollary 1, for any $\phi > 0$ $\Theta_k^m(\tilde{k}) < \Theta_k^p(\tilde{k}; \phi)$ and $\varphi^m(\tilde{k}) > \varphi^p(\tilde{k}; \phi)$. Thus, by continuity of functions $\Theta_k(\cdot)$ and $\varphi(\cdot)$, for any $\epsilon > 0$, there exist $\delta^\circ < \bar{\delta}$ and $\hat{\phi} > 0$, such that $\left| \Theta_k^p(\tilde{k}; \hat{\phi}) - \Theta_k^o(\tilde{k}^*; \delta^\circ) \right| < \epsilon$ and $\left| \varphi^p(\tilde{k}; \hat{\phi}) - \varphi^o(\tilde{k}^*; \delta^\circ) \right| < \epsilon$. ■

Proposition (4). See proof of Proposition 3 under the balanced budget condition given by Eq. (5). ■

Proposition (5). Let us consider a finite horizon economy, which ends at time $T = 2$. Parties are ideologically heterogenous. In the last period the political maximization program for each party i_t is equal to:

$$\max_{z_{i_2}} - (\psi^1 + \psi^2) l(z_{i_2} - \bar{z}_i) + \frac{\psi^2}{\psi^1} u(\mathcal{C}_{i_2}^2) + u(\mathcal{C}_{i_2}^1)$$

If $\bar{z}_{\mathfrak{R}} \neq \bar{z}_{\mathcal{L}}$ the two parties have a different objective function and in equilibrium they propose two different fiscal platforms, $z_{\mathfrak{R}_2} \neq z_{\mathcal{L}_2}$. Consequently, $\Delta u(\mathcal{C}_2^2) \neq \Delta u(\mathcal{C}_2^1)$ with both terms different from zero. Using Eq. (23) it follows that $\bar{\theta}_2 \neq 0$. Specifically it is equal to:

$$\bar{\theta}(k_2, h_2) = \frac{1}{\psi^1 + \psi^2} \left(\frac{\psi^2}{\psi^1} \Delta u(\mathcal{C}_2^2) + \Delta u(\mathcal{C}_2^1) \right) \quad (50)$$

As a result, the probability that in the last period party \mathfrak{R} and \mathcal{L} wins the election is equal to $\Pr(\theta_2 > \bar{\theta}_2) = \frac{1}{2} - \theta \bar{\theta}(k_2, h_2)$ and $\Pr(\theta_2 < \bar{\theta}_2) = \frac{1}{2} + \theta \bar{\theta}(k_2, h_2)$, respectively.

At time $T = 1$ the maximization program then becomes:

$$\begin{aligned} \max_{z_{i_1}} & - (\psi^1 + \psi^2) l(z_{i_1} - \bar{z}_i) + \frac{\psi^2}{\psi^1} u(\mathcal{C}_{i_1}^2) + u(\mathcal{C}_{i_1}^1) \\ & + \beta E [\mathcal{P}_{i_1} (W_{i_1}^2 + \theta_2 + \psi_{j_2}^2) + (1 - \mathcal{P}_{i_1}) V_{i_1}^2] \end{aligned}$$

where $\mathcal{P}_{\mathfrak{R}_1} = \frac{1}{2} - \theta \bar{\theta}(k_2, h_2)$ and $\mathcal{P}_{\mathcal{L}_1} = \frac{1}{2} + \theta \bar{\theta}(k_2, h_2)$ with $k_2 = \mathcal{K}(f_{i_1}, b_{i_1}, h_1, k_1)$ and $h_2 = H(f_{i_1}, h_1)$. Given that $\bar{z}_{\mathfrak{R}} \neq \bar{z}_{\mathcal{L}}$ then $z_{\mathfrak{R}_1} \neq z_{\mathcal{L}_1}$ and $\mathcal{P}_{\mathfrak{R}_1} \neq \mathcal{P}_{\mathcal{L}_1}$. The endogenous threshold $\bar{\theta}(k_1, h_1)$ turns out to be different from zero and a function of next period $\bar{\theta}(k_2, h_2)$:

$$\begin{aligned} \bar{\theta}(k_1, h_1) &= \frac{1}{\psi^1 + \psi^2} \left(\frac{\psi^2}{\psi^1} \Delta u(\mathcal{C}_1^2) + \Delta u(\mathcal{C}_1^1) \right) \\ &+ \frac{1}{\psi^1 + \psi^2} \beta \left(\begin{array}{l} E[\mathcal{P}_{\mathcal{L}_1} (W_{\mathcal{L}_1}^2 + \theta_2 + \psi_{j_2}^2) + (1 - \mathcal{P}_{\mathcal{L}_1}) V_{\mathcal{L}_1}^2] \\ - E[\mathcal{P}_{\mathfrak{R}_1} (W_{\mathfrak{R}_1}^2 + \theta_2 + \psi_{j_2}^2) + (1 - \mathcal{P}_{\mathfrak{R}_1}) V_{\mathfrak{R}_1}^2] \end{array} \right) \end{aligned}$$

Plugging the corresponding functions for $\mathcal{P}_{\mathfrak{R}_1}$ and $\mathcal{P}_{\mathcal{L}_1}$ and rearranging we obtain:

$$\begin{aligned} \bar{\theta}(k_1, h_1) &= \frac{1}{\psi^1 + \psi^2} \left(\frac{\psi^2}{\psi^1} \Delta u(\mathcal{C}_1^2) + \Delta u(\mathcal{C}_1^1) \right) \\ &+ \frac{\beta}{\psi^1 + \psi^2} \frac{1}{2} \left((W_{\mathcal{L}_1}^2 + V_{\mathcal{L}_1}^2) - (W_{\mathfrak{R}_1}^2 + V_{\mathfrak{R}_1}^2) \right) \\ &+ \frac{\beta \theta}{\psi^1 + \psi^2} \left[\begin{array}{l} \bar{\theta}(\mathcal{K}(f_{\mathcal{L}_1}, b_{\mathcal{L}_1}, h_1, k_1), H(f_{\mathcal{L}_1}, h_1)) (W_{\mathcal{L}_1}^2 - V_{\mathcal{L}_1}^2) \\ + \bar{\theta}(\mathcal{K}(f_{\mathfrak{R}_1}, b_{\mathfrak{R}_1}, h_1, k_1), H(f_{\mathfrak{R}_1}, h_1)) (W_{\mathfrak{R}_1}^2 - V_{\mathfrak{R}_1}^2) \end{array} \right] \end{aligned}$$

where $\bar{\theta}(k_2, h_2)$ is given by Eq. (50). The argument above can be replicated for each t . As a

result, the parties maximization program turns out to be equal to:

$$\begin{aligned} \max_{\{b_{i_t}, f_{i_t}, h_{t+1}, k_{t+1}\}} & - (\psi^1 + \psi^2) l(z_{i_t} - \bar{z}_i) \\ & + \phi u(C_{i_t}^2(b_{i_t}, h_t, k_t)) + u(C_{i_t}^1(f_{i_t}, b_{i_t}, h_t, k_t, k_{t+1})) \\ & + \beta E [\mathcal{P}_{i_t} (W_{i_t}^2 + \theta_{t+1} + \psi_{j_{t+1}}^2) + (1 - \mathcal{P}_{i_t}) V_{i_t}^2] \end{aligned}$$

under the constraint $u_c(C_{i_t}^1) - \beta E^{\mathcal{P}_{i_t}} [\Theta_{kt+1} u_c(C_{i_{t+1}}^2)] = 0$. ■

References

- [1] Alesina, A., and Tabellini, G., 1990, A Positive Theory of Fiscal Deficits and Government Debt, *Review of Economic Studies*, 57 (3), 403-414.
- [2] Azariadis, C., and Galasso, V., 2002, Fiscal Constitutions, *Journal of Economic Theory*, 103 (2), 255-281.
- [3] Azzimonti, M., 2011, Barriers to Investment in Polarized Societies, *American Economic Review*, Forthcoming.
- [4] Barro, R., 1991, Economic Growth in a Cross Section of Countries, *The Quarterly Journal of Economics*, 106 (2), 407-443.
- [5] Boldrin, M., and Montes, A., 2005, The Intergenerational State Education and Pension, *Review of Economic Studies*, 72 (3), 651-664.
- [6] Docquier, F., Paddison, O., and Pestieau, P., 2006, Optimal Accumulation in an Endogenous Growth Setting with Human Capital, *Journal of Economic Theory*, 134, 361-378.
- [7] Forni, L., 2005, Social Security as Markov Equilibrium in OLG Models, *Review of Economic Dynamics*, 8 (1), 178-194.
- [8] Gonzalez-Eiras, M., and Niepelt, D., 2011, Aging, Government Budgets, Retirement, and Growth, *CESifo Working Paper*.
- [9] Klein, P., Krusell, P., and Rios-Rull, J. V., 2008, Time Consistent Public Expenditures, *Review of Economic Studies*, 75 (3), 789-808.
- [10] Krusell, P., Quadrini, V., and Ríos-Rull, J. V., 1997, Politico-Economic Equilibrium and Economic Growth, *Journal of Economic Dynamics and Control*, 21 (1), 243-272.
- [11] Lindbeck, A., and Weibull, J., 1987, Balanced-Budget Redistribution as the Outcome of Political Competition, *Public Choice*, 52, 273-297.
- [12] Samuelson, P. A., 1958, An Exact Consumption-Loan Model of Interest with and without the Social Contrivance of Money, *Journal of Political Economy*, 66 (6), 467-482.
- [13] Song, Z., Storesletten, K., and Zilibotti, F., 2011, Rotten Parents and Disciplined Children: A Politico-Economic Theory of Public Expenditure and Debt, mimeo.