Optimal Control of Nonlinear Econometric Models with Rational Expectations

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Abstract

In this paper, we develop the OPTCONRE algorithm, which provides approximate numerical solutions to optimal control problems with a quadratic objective function for nonlinear econometric models with rational expectations. In such models some variables are forward-looking instead of having the usual 'causal' time structure of models with backward-looking or static expectations. The algorithm, which was programmed in MATLAB, allows for deterministic and stochastic control, the latter with open-loop and passive learning information patterns. We use a small quarterly macroeconometric model for Slovenia to illustrate the applicability of the algorithm. We examine differences in the optimal policy design between a model alternative with rational and with static expectations about the inflation rate. This shows the convergence of the OPTCONRE algorithm and its practical usefulness for problems of stabilization policy in small-sized macroeconometric models.

Keywords optimal control; rational expectations; econometric modeling; policy applications

1 Introduction

Optimal control problems are encountered in many areas of science from engineering to economics. In particular, there have been many studies on determining optimal policies for management and economic models. One of the main questions in economics is: which strategy should be used in order to influence certain economic variables such as unemployment, inflation or GDP and to bring them to the desired values. For example, a policy maker responsible for fiscal policy in a country may determine tax rates and government expenditures such as to obtain the most desired (ideal) time paths for such objective variables as the rate of growth of real GDP, the rate of unemployment or the rate of inflation, among others. Another example for optimal control from business field is that, when the manager of a firm may want to maximize profits or minimize costs subject to the restrictions posed by the firms current production technology or available personnel, etc. To answer such questions optimal control theory is applied, which has been used intensively for calculating optimal policy paths with macroeconometric models of the Cowles Commission and related type. Unfortunately it hence did not incorporate so-called non-causal relations, that is, forward-looking variables such as rationally expected prices. This is a disadvantage because it is very important to understand for the dynamic processes what kind of information and how it is put together to frame an estimate of future conditions. Taking the forward-looking variables in the system into account makes the solving techniques more reliable and precise.

Only recently endeavours have started to incorporate rational expectations into optimal control techniques. One of the most promising framework is due to Amman and Kendrick (1999, 2000, 2003), who developed algorithms for the optimal deterministic and stochastic control of dynamic economic systems containing rational expectations. But these works are restricted on linear models, which is not realistic at all for even the simplest econometric models in use today. Thus, extending their approach to nonlinear econometric models is highly desirable. In the present paper the extension of the Amman-Kendrick algorithm to nonlinear models is done. We combine the research results from Amman and Kendrick and the OPTCON algorithm (Matulka and Neck 1992, Blueschke-Nikolaeva et al. 2012), which solves optimal control problems with nonlinear dynamic models. As a result an algorithm called OPTCONRE was created which serves to determine approximately optimal time paths for control variables in the context of a quadratic objective (cost) function and a nonlinear model with rational expectations.

In this paper, an optimal control problem to be solved approximately by OPTCONRE algorithm is introduced in Section 2. In Section 3 a brief description of the OPTCON algorithm is presented. The new OPTCONRE algorithm is described in Section 4. Section 5 shows the results of an application of the OPTCONRE algorithm to a small econometric model for Slovenia. The application experiment demonstrates that the algorithm works, converges and delivers results in plausible optimal policies. Because the model has a simple structure, the results are very close to those obtained from a related model without rational expectations, but it is assumed that this property does not hold generally. Section 7 concludes.

2 The optimal control problem

The OPTCONRE algorithm was designed to determine approximate solutions to optimum control problems with a quadratic objective function (a loss function to be minimized), which is formulated in quadratic tracking form and is written as

$$J = E\left[\sum_{t=1}^{T} L_t(x_t, u_t)\right], \tag{1}$$

with

$$L_t(x_t, u_t) = \frac{1}{2}\begin{pmatrix} x_t - \tilde{x}_t \\ u_t - \tilde{u}_t \end{pmatrix}' W_t \begin{pmatrix} x_t - \tilde{x}_t \\ u_t - \tilde{u}_t \end{pmatrix} \tag{2}$$

and a nonlinear multivariate discrete-time dynamic system under additive and parameter uncertainties, which has the form

$$x_t = f(x_{t-1}, x_t, u_t, Ex_t, \theta, z_t) + \varepsilon_t, \quad t = 1, ..., T, \tag{3}$$

where x_t is an n-dimensional vector of state variables that describes the state of the economic system at any point in time t. u_t is an m-dimensional vector of control variables, $\tilde{x}_t \in R^n$ and $\tilde{u}_t \in R^m$ are given 'ideal' (desired, target) levels of the state and control variables respectively. T denotes the terminal time period of the finite planning horizon. W_t is an $((n+m)\times(n+m))$ matrix, specifying the relative weights of the state and control variables in the objective function. In a frequent special case, W_t is a matrix including a discount factor α with $W_t = \alpha^{t-1}W$. W_t (or W) is symmetric. Ex_t represents the rational expectations of the state variables. Ex_t is an $n \times \tau$ matrix, with non-zero rows for variables with rational expectations and τ denoting the maximum lead in the expectations formation. θ is a p-dimensional vector of parameters whose values are assumed to be constant but may be unknown to the decision-maker (parameter uncertainty), z_t denotes an l-dimensional vector of non-controlled exogenous variables, and ε_t is an n-dimensional vector of additive disturbances (system error). θ and ε_t are assumed to be independent random vectors with expectations $\hat{\theta}$ and O_n respectively and covariance matrices $\Sigma^{\theta\theta}$ and $\Sigma^{\varepsilon\varepsilon}$ respectively. f is a vector-valued function, $f^i(.....)$, is the i-th component of $f(.....)$, $i = 1, ..., n$.

3 The OPTCON algorithm

The OPTCON algorithm allows to calculate numerical solutions to the class of optimum control problems described in Section 2 without rational expectations. It combines elements of previous algorithms developed by Chow (1975, 1981), which incorporate nonlinear systems but no multiplicative uncertainty, and Kendrick (1981), which deal with linear systems and all kinds of uncertainty. The first version of the algorithm, OPTCON1, which considers deterministic and stochastic open-loop solutions, is described in detail in Matulka and Neck (1992). The second version, OPTCON2, allows to take stochastic passive learning strategies into account as well and is described in Blueschke-Nikolaeva et al. (2012).

The nonlinearity problem is tackled iteratively in the OPTCON2 algorithm. Following procedure is done repeatedly. First, the nonlinear problem is linearized, then this linear approximation is solved and the linear solution is taken as a new tentative path. If a convergence criterion is fulfilled, the solution of the last iteration is taken as the optimal solution to the nonlinear problem and the algorithm stops. It should be also mentioned that the nonlinear system is solved in each step using one of the alternative methods, namely the Newton-Raphson, Gauss-Seidel, Levenberg-Marquardt or trust region methods.

4 The OPTCONRE algorithm

In the present work we extend the OPTCON2 algorithm for models with rational expectations. By doing this we follow the framework presented in Amman and Kendrick (1999, 2000, 2003). The main difference is that we deal with nonlinear problems which are described by equation (3):

$$x_t = f(x_{t-1}, x_t, u_t, Ex_t, \theta, z_t) + \varepsilon_t, \ \ t = 1, ..., T,$$

where Ex_t ($E_{t-1}x_{t+j}, \ \ j = 0, ..., \tau{-}1$) is the matrix of expected state variables for time periods $t + j$ expected at time $t - 1$. Ex_t is an $n \times \tau$ matrix, but only rows representing forward-looking expectations variables (variables with rational expectations) are non-zero. τ denotes the maximum lead in the expectations formation (to be more precise, the maximum lead is τ-1 as shown in equation (4)). We use the same iterative structure to approximate the nonlinear problem as in previous versions of OPTCON, which means that we repeatedly linearize the system, solve it and take the current solution as the new tentative path for the nonlinear problem. By linearizing[1] we transform the autonomous nonlinear system (3) to the following time-varying linear form:

$$x_t = A_t x_{t-1} + B_t u_t + C_t z_t + \sum_{j=0}^{\tau-1} D_{jt} E_{t-1} x_{t+j} + \phi_t + \xi_t. \tag{4}$$

Using Sims' method (Sims (2002)) we transform (4) in the form

$$\Gamma_{0t}\tilde{x}_t = \Gamma_{1t}\tilde{x}_{t-1} + \Gamma_{2t}u_t + \Gamma_{3t}z_t + \Gamma_{4t}\phi_t + \Gamma_5\xi_t, \tag{5}$$

where

$$\Gamma_{0t} = \begin{bmatrix} I - D_{1t} & -D_{2t} & \cdots & -D_{\tau-2t} & -D_{\tau-1t} \\ I & 0 & \cdots & 0 & 0 \\ 0 & I & \cdots & 0 & 0 \\ \vdots & & \ddots & 0 & 0 \\ 0 & \cdots & & I & 0 \end{bmatrix}, \Gamma_{1t} = \begin{bmatrix} A_t & 0 & \cdots & 0 \\ 0 & I & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \ddots & & I \end{bmatrix},$$

$$\Gamma_{2t} = \begin{bmatrix} B_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \Gamma_{3t} = \begin{bmatrix} C_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \Gamma_{4t} = \Gamma_{5t} = \begin{bmatrix} I \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

[1]To initialize the linearization we need tentative paths for the expected state variables. In that case the given values of the state variables in the previous time period are taken as the tentative paths.

and the augmented state vector $\tilde{x}_t = \begin{bmatrix} x_t \\ Ex_{t+1} \\ Ex_{t+2} \\ \vdots \\ Ex_{t+\tau-1} \end{bmatrix}$.

We apply the QZ decomposition to the system matrices Γ_{0t} and Γ_{1t}:

$$\begin{aligned} \Lambda_t &= Q_t \Gamma_{0t} Z_t, \\ \Omega_t &= Q_t \Gamma_{1t} Z_t, \end{aligned} \tag{6}$$

where Λ and Ω are upper triangular matrices and $\forall i \; \omega_{ii}/\lambda_{ii}$ are the generalized eigenvalues.

This QZ decomposition allows us to obtain the generalized eigenvalues and to transform equation (5) to the form:

$$\Lambda \omega_t = \Omega_1 \omega_{t-1} + Q\Gamma_2 u_t + Q\Gamma_3 z_t + Q\Gamma_4 \phi_t + Q\Gamma_5 \xi_t. \tag{7}$$

Using the triangular structure of Λ and Ω, equation (7) can be rewritten as:

$$\begin{aligned} &\begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ 0 & \Lambda_{22} \end{bmatrix} \begin{bmatrix} \omega_{1,t} \\ \omega_{2,t} \end{bmatrix} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & \Omega_{22} \end{bmatrix} \begin{bmatrix} \omega_{1,t-1} \\ \omega_{2,t-1} \end{bmatrix} + \\ &\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \Gamma_2 u_t + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \Gamma_3 z_t + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \Gamma_4 \phi_t + \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \Gamma_5 \xi_t \end{aligned} \tag{8}$$

with unstable eigenvalues in the lower right corner, i.e. in the matrices Λ_{22} and Ω_{22}. This allows us to derive $\omega_{2,t}$ as a function of future instruments and exogenous variables:

$$\gamma_t = \omega_{2,t} = -\sum_{j=0}^{\infty} \tilde{M}_{tj} \Omega_{22t+j}^{-1} Q_{2t+j} (\Gamma_{2t+j} u_{t+j} + \Gamma_{3t+j} z_{t+j} + \Gamma_{4t+j} \phi_{t+j} + \Gamma_{5t+j} \xi_{t+j}), \tag{9}$$

where $\tilde{M}_{tj} = \prod_{i=0}^{j-1} (\Omega_{22t+i}^{-1} \Lambda_{22t+i})$ for $j > 0$ and $\tilde{M}_{tj} = I$ for $j = 0$.

Inserting equation (9) into equation (7) gives us:

$$\tilde{\Lambda}\omega_t = \tilde{\Omega}\omega_{t-1} + \tilde{\Gamma}_2 u_t + \tilde{\Gamma}_3 z_t + \tilde{\Gamma}_4 \phi_t + \tilde{\Gamma}_5 \xi_t + \tilde{\gamma}_t \tag{10}$$

with $\tilde{\Lambda} = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} \\ 0 & I \end{bmatrix}$, $\tilde{\Omega} = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ 0 & 0 \end{bmatrix}$, $\tilde{\gamma}_t = \begin{bmatrix} 0 \\ \gamma_t \end{bmatrix}$, $\tilde{\Gamma}_2 = \begin{bmatrix} Q_1 \\ 0 \end{bmatrix} \Gamma_2$, $\tilde{\Gamma}_3 = \begin{bmatrix} Q_1 \\ 0 \end{bmatrix} \Gamma_3$, $\tilde{\Gamma}_4 = \begin{bmatrix} Q_1 \\ 0 \end{bmatrix} \Gamma_4$, $\tilde{\Gamma}_5 = \begin{bmatrix} Q_1 \\ 0 \end{bmatrix} \Gamma_5$.

Using $\tilde{x}_t = Z'\omega_t$ we rewrite equation (10) as

$$\tilde{x}_t = \tilde{A}_t \tilde{x}_{t-1} + \tilde{B}_t u_t + \tilde{C}_t \tilde{z}_t + \tilde{\phi}_t + \tilde{\xi}_t, \tag{11}$$

where $\tilde{A}_t = Z_t \tilde{\Lambda}_t^{-1} \tilde{\Omega}_t Z_t'$, $\tilde{B}_t = Z_t \tilde{\Lambda}_t^{-1} \tilde{\Gamma}_{2t}$, $\tilde{C}_t = [Z_t \tilde{\Lambda}_t^{-1} \tilde{\Gamma}_{3t} \ Z_t \tilde{\Lambda}_t^{-1}]$, $\tilde{z}_t = \begin{bmatrix} z_t \\ \tilde{\gamma}_t \end{bmatrix}$, $\tilde{\phi}_t = Z_t \tilde{\Lambda}_t^{-1} \tilde{\Gamma}_{4t} \phi_t$, $\tilde{\xi}_t = Z_t \tilde{\Lambda}_t^{-1} \tilde{\Gamma}_{5t} \xi_t$ and $\tilde{\Lambda}^{-1} = \begin{bmatrix} \Lambda_{11}^{-1} & -\Lambda_{11}^{-1}\Lambda_{12} \\ 0 & I \end{bmatrix}$.

Then we can apply the LQ optimal control framework from OPTCON to equation (11) using Bellman's dynamic programming.[2]

[2] We skip the detailed description of these equations which combine those used in OPTCON2 (see Blueschke-Nikolaeva et al. (2012)) and the ones described in Amman and Kendrick (2003).

5 An application

OPTCONRE, the extended version of the OPTCON2 algorithm which takes rational expectations into account was implemented in MATLAB. In order to test its convergence[3], a relatively simple and small macroeconometric model for Slovenia was used. The details of this model are presented in Section 5.1. Section 5.2 gives the optimization results for this macroeconometric model.

5.1 The SLOVNLRE model

In this work we extend the small nonlinear macroeconometric model of the Slovenian economy, called SLOVNL, which was presented in Blueschke-Nikolaeva et al. (2012), by adding rational expectations for one of the state variables. The model is estimated using the quarterly data for the time periods 1995:1 to 2006:4. The start period for the optimization is 2004:1 and the end period is 2006:4 (12 periods).

Variables used in SLOVNL

Endogenous (state) variables:

[3] In a first implementation test, we successfully reproduced the results of Amman and Kendrick (2003) for a very simple one state, one control linear model.

$x[1]:$	CR	real private consumption
$x[2]:$	$INVR$	real investment
$x[3]:$	$IMPR$	real imports of goods and services
$x[4]:$	$STIRLN$	short term interest rate
$x[5]:$	$GDPR$	real gross domestic product
$x[6]:$	VR	real total aggregate demand
$x[7]:$	PV	general price level
$x[8]:$	$Pi4$	rate of inflation

Control variables:

u[1]	$TaxRate$	net tax rate
u[2]	GR	real public consumption
u[3]	$M3N$	money stock, nominal

Exogenous non-controlled variables:

z[1]	EXR	real exports of goods and services
z[2]	$IMPDEF$	import price level
z[3]	$GDPDEF$	domestic price level
z[4]	$SITEUR$	nominal exchange rate SIT/EUR

SLOVNL model equations:

The first four equations are estimated by FIML, the last four equations are identities.[4]

$$\begin{array}{ccccccccc} CR_t & = & 240.9398 & + & 0.740333 & CR_{t-1} & + & 0.111727 & GDPR_t \quad (1 - \frac{TaxRate_t}{100}) \\ & & (189.7449) & & (0.1115) & & & (0.0330) & \end{array}$$

[4]Standard deviations are given in brackets.

$$
\begin{array}{lllllll}
- & \underset{(2.5848)}{1.007353} & (STIRLN_t - Pi4_t) & - & \underset{(2.4966)}{4.773533} & Pi4_t
\end{array}
$$

$$
\begin{aligned}
INVR_t = \;& \underset{(176.8549)}{75.41731} + \underset{(0.1423)}{0.932211}\; INVR_{t-1} + \underset{(0.0924)}{0.264523}\; (VR_t - VR_{t-1}) \\
& - \underset{(6.9044)}{0.455511}\; (STIRLN_t - Pi4_t) - \underset{(3.1277)}{2.981241}\; Pi4_t
\end{aligned}
$$

$$
IMPR_t = IMPR_{t-1} + \underset{(0.0724)}{0.826449}\; (VR_t - VR_{t-1}) - \underset{(18.9336)}{38.14954}\; SITEUR_t
$$

$$
\begin{aligned}
STIRLN_t = \;& \underset{(0.1375)}{0.811606}\; STIRLN_{t-1} - \underset{(0.0008)}{0.000877}\; \frac{(M3N)_t}{PV_t} \cdot 100 \\
& + \underset{(0.0026)}{0.002746}\; GDPR_t
\end{aligned}
$$

$$
GDPR_t = CR_t + INVR_t + GR_t + EXR_t - IMPR_t
$$

$$
VR_t = GDPR_t + IMPR_t
$$

$$
PV_t = \frac{GDPR_t}{VR_t} \cdot GDPDEF_t + \frac{IMPR_t}{VR_t} \cdot IMPDEF_t
$$

$$
Pi4_t = \frac{PV_t - PV_{t-4}}{PV_{t-4}} \cdot 100
$$

In the present paper we replace the current values of the inflation variable $Pi4$ in the first two equations (which can be interpreted as static expectations of next-period's inflation rate) by the expected values which are formed rationally. The adjusted equations for consumption and investment, which are used in the SLOVNLRE model, looks then as follows:

$$\begin{array}{rclclclcl} CR'_t & = & \underset{(189.7449)}{240.9398} & + & \underset{(0.1115)}{0.740333} & CR_{t-1} & + & \underset{(0.0330)}{0.111727} & GDPR_t \quad (1 - \frac{TaxRate_t}{100}) \\ & - & \underset{(2.5848)}{1.007353} & & (STIRLN_t - \mathbf{Pi4^e_t}) & - & \underset{(2.4966)}{4.773533} & \mathbf{Pi4^e_t} & \end{array}$$

$$\begin{array}{rclclclcl} INVR'_t & = & \underset{(176.8549)}{75.41731} & + & \underset{(0.1423)}{0.932211} & INVR_{t-1} & + & \underset{(0.0924)}{0.264523} & (VR_t - VR_{t-1}) \\ & - & \underset{(6.9044)}{0.455511} & & (STIRLN_t - \mathbf{Pi4^e_t}) & - & \underset{(3.1277)}{2.981241} & \mathbf{Pi4^e_t} & \end{array}$$

This means that the Ex matrix as stated in equation (4) contains only one non-zero row, namely the eighth row which corresponds to the inflation variable.

The objective function penalizes deviations of objective variables from their target values according to equations (1) and (2):

$$J = E\left[\sum_{t=1}^{T} L_t(x_t, u_t)\right],$$

with

$$L_t(x_t, u_t) = \frac{1}{2}\begin{pmatrix} x_t - \tilde{x}_t \\ u_t - \tilde{u}_t \end{pmatrix}' W_t \begin{pmatrix} x_t - \tilde{x}_t \\ u_t - \tilde{u}_t \end{pmatrix}.$$

The ‘ideal’ values of the state and control variables ($\tilde{x}_t$ and $\tilde{u}_t$ respectively) and the corresponding weights are chosen in the same way as in Blueschke-Nikolaeva et al. (2012).

Next, the OPTCON2 algorithm is applied to the SLOVNL model and the OPTCONRE algorithm is applied to the SLOVNLRE model in order to determine approximately optimal fiscal and monetary policies. By running this experiment we expect to get two different insights. First, we want to check the applicability and the convergence of the OPTCONRE algorithm for a model with rational expectations under a nonlinear optimal control problem. Second, we want to figure out the impact of introducing rational expectations on optimal policies. Using two sister models, SLOVNL and SLOVNLRE, allows us to track the output differences as a result of the rational expectations.

5.2 Results

In this subsection we present the optimal control (and non-controlled simulation) results of applying the OPTCON2 algorithm to the SLOVNL model and the OPTCONRE algorithm to the SLOVNLRE model. In the following we present graphical results for three control variables, $TaxRate, GR, M3N$, and five state variables, $CR, INVR, STIRLN, GDPR, Pi4$. Each of the figures contains four different trajectories:

* *optcon2*: shows the results of the OPTCON2 algorithm applied to the SLOVNL model
* *optcon2*_re: shows the results of the OPTCONRE algorithm applied to the SLOVNLRE model
* *simulat*: shows the uncontrolled simulation results

* *target*: gives the target ('ideal') values

The following figures 1 - 3 show the results of the control variables.

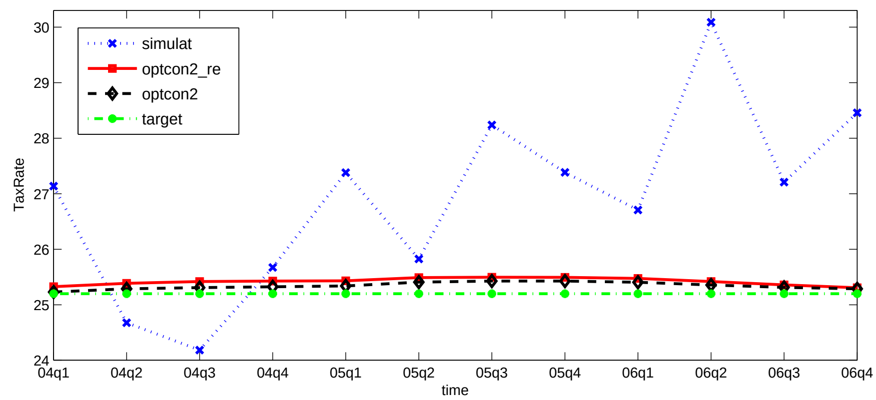

Figure 1: net tax rate (TaxRate)

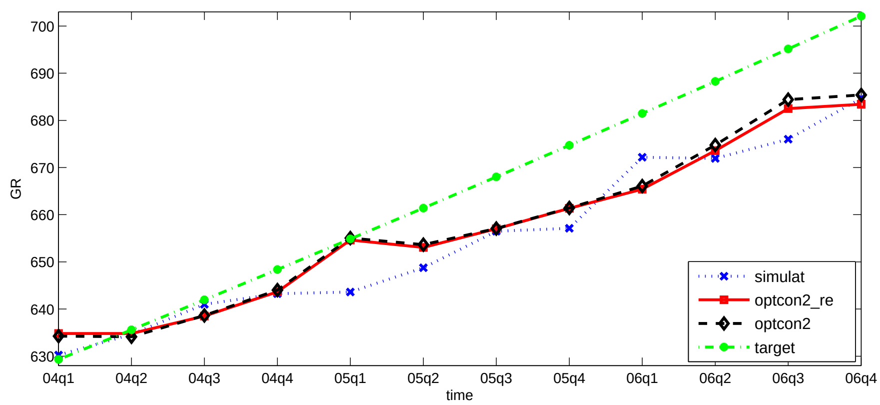

Figure 2: real public consumption (GR)

The differences between the experiments with and without rational expectations are extremely small. Both time paths under *optcon2* and *optcon2_re* nearly coincide. We can only observe very small differences for the controls

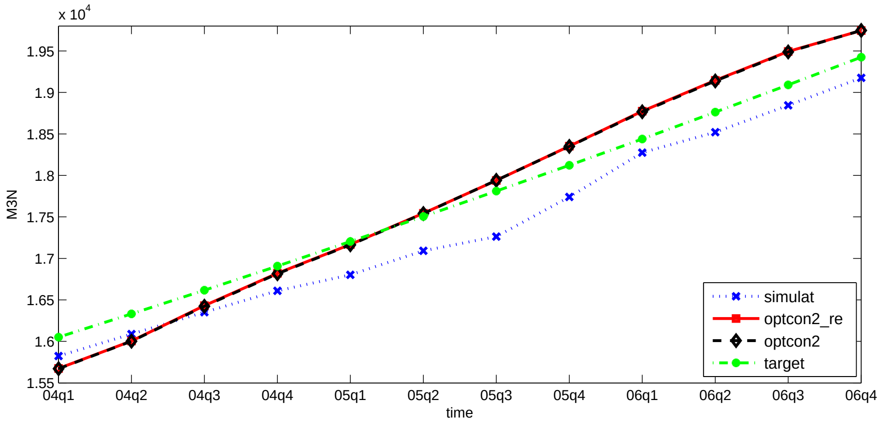

Figure 3: money stock, nominal (M3N)

$TaxRate$ and GR. In both cases, the solution for the model with rational expectations suggests to run a slightly more restrictive fiscal policy.

Figures 4 - 8 show the results for the state variables.

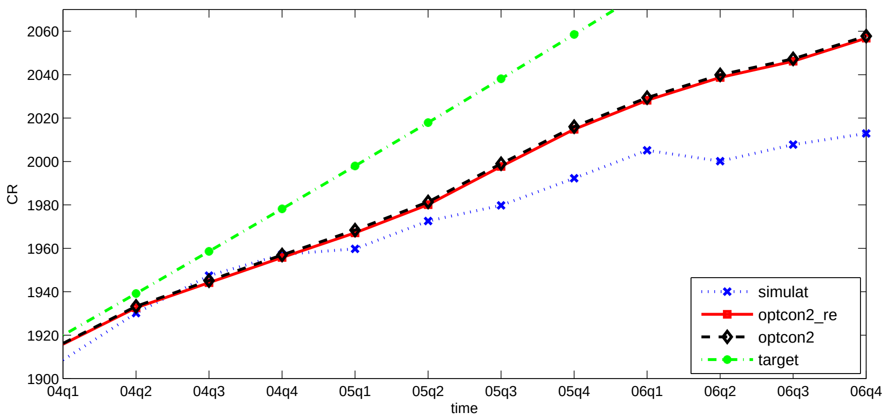

Figure 4: real private consumption (CR)

Also for the state variables the graphical differences between the experiments with and without rational expectations are very small. This means that for the nonlinear problem under consideration, the introduction of ra-

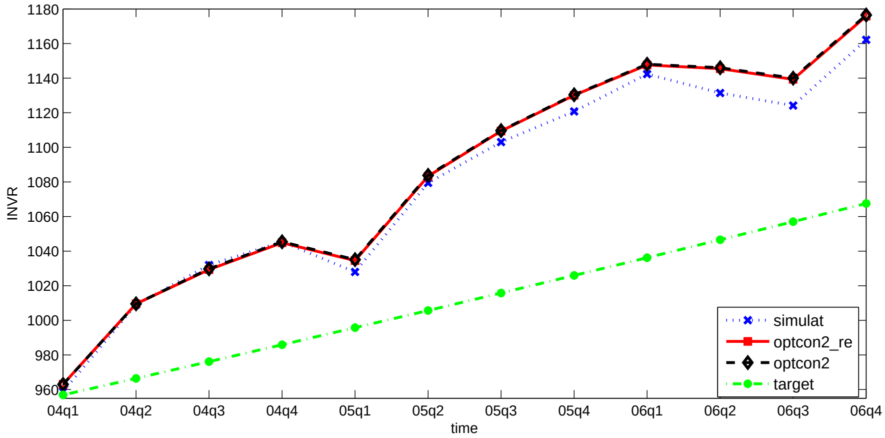

Figure 5: real investment (INVR)

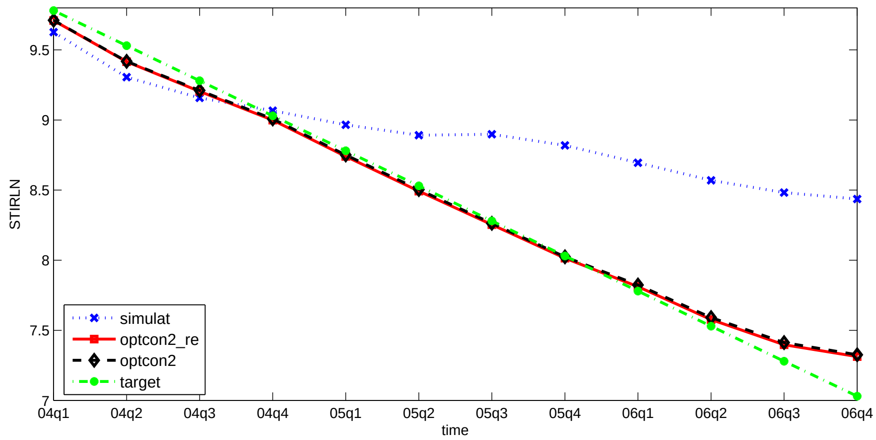

Figure 6: short term interest rate (STIRLN)

tional expectations for the inflation variable has nearly no effect. This observation is supported by looking at the objective values. The initial objective value which is calculated from the non-controlled simulation (using historical values of the exogenous variables) is 2,759,743. The objective value of the OPTCON2 solution is 904,650and the objective value of the OPT-

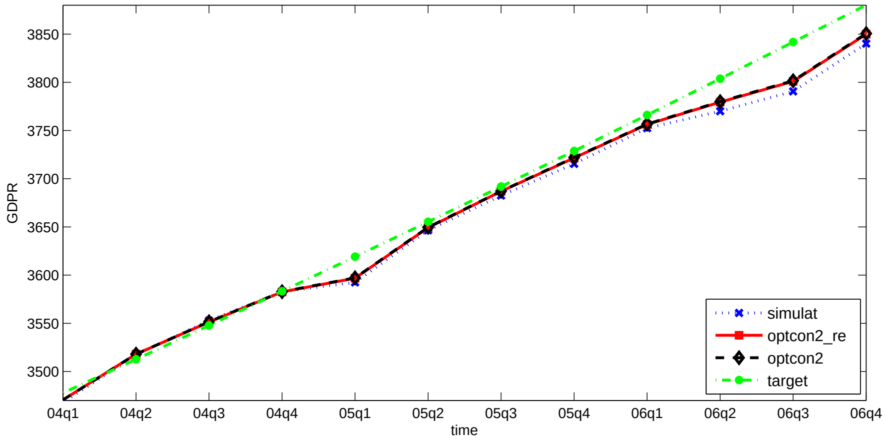

Figure 7: real gross domestic product (GDPR)

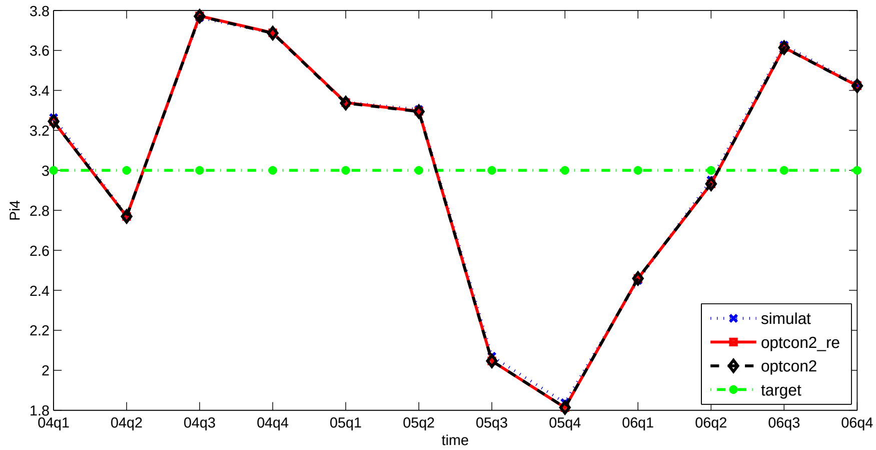

Figure 8: rate of inflation (Pi4)

CONRE solution is 906,702 which is very close to the experiment without rational expectations. One reason for this result is the fact that in the simple econometric model used, the price level and hence the inflation rate are mostly determined by exogenous non-controlled variables and do not depend heavily on variables affected by the controls. This implies that a rational policy-maker will not gain much by sophisticated techniques of estimating future inflation under this model and cannot improve upon using the current inflation rate as predictor for next-period's inflation rate.

6 Conclusion

In this paper, we have presented the OPTCONRE algorithm, which determines approximately optimal trajectories of policy instruments in dynamic optimization problems with a quadratic intertemporal objective function under a nonlinear discrete-time economic model where some variables includes rational expectations. The algorithm can be regarded as an extension of the OPTCON / OPTCON2 algorithm to non-causal dynamic systems or as an extension of Amman's and Kendrick's algorithm for rational expectations models to models with nonlinearities. Applying OPTCONRE to a small and simple econometric model of the Slovenian economy, we showed that the algorithm and its implementation in MATLAB works, i.e. converges and yields plausible results. These results (in the special case of a linear model) coincide with results obtained by Amman and Kendrick. In the particular model used, we found that the introduction of rational instead of static expectations did not change substantially the results obtained for a related optimization problem with the analogous model employing static expectations. Further research will have to show how the introduction of rational expectations changes optimal policies in more sophisticated and realistic models. In any case, the OPTCONRE algorithm can be regarded as a useful tool to be employed for a large class of econometric and related models such as DSGE models which routinely include rational expectations of some or all endogenous variables.

References

Amman, H. M., Kendrick, D. A., 1999. Linear-quadratic optimization for models with rational expectations. Macroeconomic Dynamics 3 (4), 534–543.

Amman, H. M., Kendrick, D. A., 2000. Stochastic policy design in a learning environment with rational expectations. Journal of Optimization Theory and Applications 105 (3), 509–520.

Amman, H. M., Kendrick, D. A., 2003. Mitigation of the lucas critique with stochastic control methods. Journal of Economic Dynamics and Control 27, 2035–2057.

Blueschke-Nikolaeva, V., Blueschke, D., Neck, R., 2012. Optimal control of nonlinear dynamic econometric models: An algorithm and an application. Computational Statistics and Data Analysis 56 (11), 3230–3240.

Chow, G. C., 1975. Analysis and Control of Dynamic Economic Systems. John Wiley & Sons, New York.

Chow, G. C., 1981. Econometric Analysis by Control Methods. John Wiley & Sons, New York.

Kendrick, D. A., 1981. Stochastic Control for Economic Models. McGraw-Hill, New York. Second edition, http://www.utexas.edu/cola/_db/app/file.php?id=495395.

Matulka, J., Neck, R., 1992. OPTCON: An algorithm for the optimal control of nonlinear stochastic models. Annals of Operations Research 37, 375–401.

Sims, C. A., 2002. Solving linear rational expectations models. Computational Economics 20, 1 – 20.